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# A PROPOSED STANDARD METHOD OF COLORIMETRY

BY
HERBERT E. IVES

The problem of the accurate measurement and specification of color is not one which has, as yet, received a generally acceptable solution. The choice is between simple and scientifically attractive methods of measurement on the one hand, which are not sufficiently accurate for technical purposes, and, on the other, methods which are undesirably complicated or rest on empirical bases. More specifically, in the first group lie the methods of measurement and specification in terms of three elements, namely, the three color additive method, the three color subtractive method, the spectrum hue, brightness and purity method. These all suffer in accuracy from the fact that their renderings differ from observer to observer by amounts greater than can be tolerated in technical color reproduction. They require, for their successful utilization, the employment of observers of carefully selected normal or average color vision or the development of methods of correction applicable to observers of known characteristics of vision.[1] In the second

[1] Assuming that the adoption of adequate methods of test and selection of observers would bring the three-element methods under serious consideration for the selection of one as a standard method, it would appear that the additive three-color (red, green and blue mixture) method presents the most advantages. The single setting used to obtain a measurement calls for an exact match, both of brightness and color, which from the photometric standpoint is ideal. The measurement when obtained can be transformed into color sensation, or spectrum hue, brightness and purity values, if so desired, by comparatively simple steps. The spectrum hue, brightness and purity method as a method of measurement suffers under the serious disadvantage that one of the several settings necessary for a complete measurement is that of brightness with a

group lie the various color charts or scales with their hundreds of sample colors, each of which must ultimately be measured and specified by some standard method, and whose reliable reproduction in quantity is a stupendous practical color problem in itself.[2]

As the present writer pointed out in a discussion of color measurement some years ago,[3] the only unique specification of a color is by means of its spectrophotometric analysis, in which paradoxically, the measurement of *color* as such does not enter. The conclusion then drawn was that the most promising generally acceptable solution of the color measurement problem was to be sought in the development of simple and accurate methods of spectrophotometry and methods of expressing spectrophotometric results.

The objections to the use of the spectrophotometer for practical color measurement are obvious. It is an expensive and delicate instrument. A large number of measurements are necessary to specify a single color. The color cannot be reproduced for study or comparison by setting the indicator on the instrument to the

---

large color difference, thus introducing a fertile field for wide discrepancies. The three-color subtractive method shares with the additive method the merit that the setting which yields the measurement is an exact match. It is, however, less simple, for the reason that the transmission, purity and hue of the absorbing "wedges" all change with depth. Unless the white light source which is modified by the absorbing wedges consists of spectrally narrow red, green and blue bands, the light transmitted by the three superposed wedges changes in intensity and purity at the same time. This necessitates the use of a neutral tint wedge or other device to complete the match, whereby the system becomes one of four elements instead of three. Furthermore, the reduction of the subtractive measurements to sensation, or hue, brightness and purity values is exceedingly difficult.

[2] An intermediate position is occupied by the polarization color scale as used in the Arons Chromoscope. Here the matter of reproducibility is well taken care of, being a question of angles of setting and thicknesses of quartz. The colors produced are spectrally different in character from the ones measured so that some difference of setting between different observers will occur, although this difference is much less than with the three color mixture method. The measurements when obtained are on scales having no obvious relation to wave-length or hue, and their reduction to a color sensation scale is a much lengthier process than the reduction of three color mixture measurements.

[3] "The Transformation of Color Mixture Equations from One System to Another." Journal Franklin Institute, Dec. 1915, p. 673.

measured values. The results—plots of readings against wave-lengths—mean little to anyone except a specially trained expert.

These and other objections may be granted at once. On the other hand, there is no ground for believing that satisfactory color measurement will ever be done by anything except high-grade and hence expensive apparatus, or that really accurate color

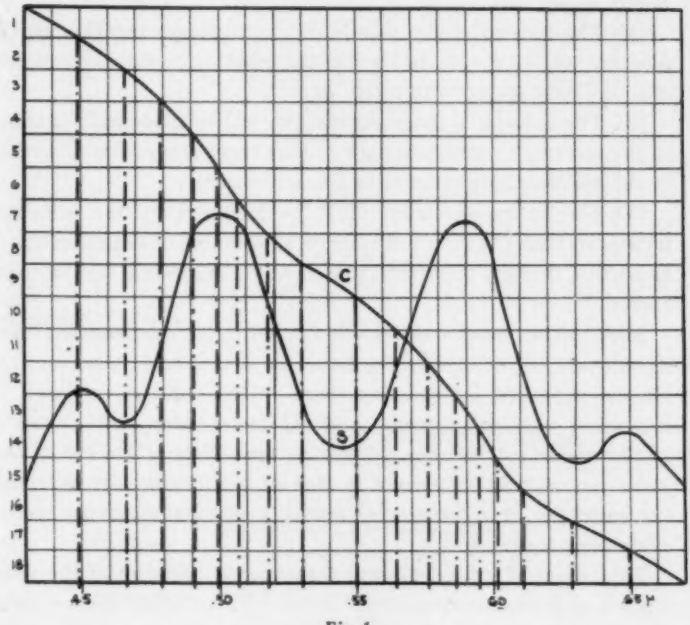



*Fig. 1*

Hue sensibility (S), and hue scale of spectrum (C). The hue scale is here shown divided into 18 equal parts, and the wave-lengths corresponding to the limits of these equal parts are obtained by projection on the wave-length axis (dot and dash lines).

specification can be made simple. If the conclusion is valid that spectrophotometric analysis is the only method of color measurement which will ultimately prove satisfactory, the obvious next step is to decide on the simplest and most practical way to work it out, having in mind that *color* rather than *spectral distribution* is the object of the measurement. This was the problem whose

study led to the method proposed below, which may be roughly described as a method in which the hue scale rather than the wave-length scale is employed and in which the whole spectrum is covered whatever the fineness of the scale used.

In considering the problem of simplifying spectrophotometric measurements for color specification, the following three requirements appeared:

(1) The measurements should be close enough together on the *hue scale* of the spectrum so that the resultant values completely and uniquely specify the color.

(2) The individual measurements should be made with portions of the spectrum narrow enough so that there is no color difference in the spectrophotometer field at each setting.

(3) The entire spectrum must be included by the measurements, so that the effect of narrow absorption or emission bands or abrupt changes of intensity with wave length will not be overlooked.

These three requirements are not entirely independent; their significance is perhaps best grasped by proceeding at once to the scheme proposed for meeting them. This consists, in brief, *in measuring adjacent patches of the spectrum, each patch being of width inversely as the hue perception sensibility of the eye at that point and sufficiently narrow so that no significant color difference can enter due to differences of intensity distribution in the spectra of the compared colors.*

The object of using adjacent (juxtaposed) patches of the spectrum is of course to meet requirement (3) obviating the necessity for measuring an infinite number of points. The choice of patches so narrow as to show no color difference in the photometric field is primarily in order to eliminate errors due to differences in color vision. Variation in width of patch with the hue sensibility (Fig. 1)[4] is in order to hold to a minimum the number of patches necessary. It will be clear that, given the instrumental means to make a spectrum measurement of the kind postulated, each setting will be a simple equality setting with no distinct color or signi-

[4] See Nutting, "A Method for Constructing the Natural Scale of Pure Color," Bull. Bureau of Standards, Vol. 6, No. 1.

ficant spectral composition difference, thus avoiding the sources of error in the three-element methods of analysis. The complete measurement will consist of the minimum number of settings by which the (subjective) color element can be unambiguously represented, thus having an advantage over an ordinarily spaced spectrophotometric series of settings, in which the number of

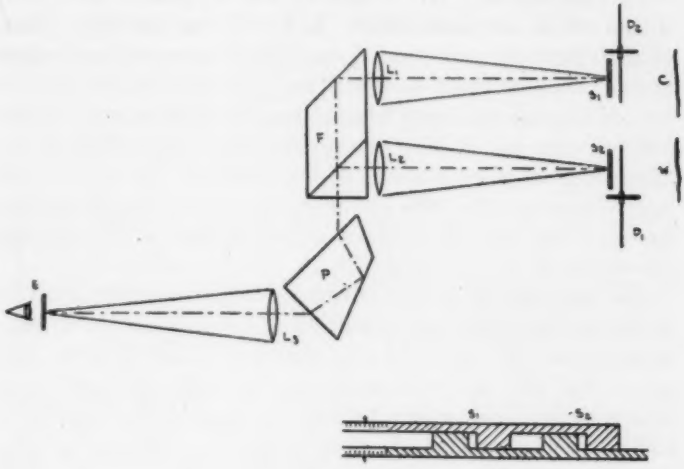



*Fig. 2*

Hue scale patch spectrophotometer
C, surface whose color is to be measured
W, standard white surface
$S_1$, $S_2$, adjustable slits
$L_1$, $L_2$, collimator lenses
F, photometric prism
P, dispersing prism
$L_3$, telescope lens
E, viewing slit.

points of setting would be larger than necessary in the region of slow hue change. Finally, since the whole spectrum eventually enters the instrument, any type of color of any extreme of spectral composition such as a line source can be measured, which is not the case with the spectrophotometer as ordinarily used with settings at isolated wave-lengths.

As an instrumental means to achieve the measurements it is proposed to utilize the principle of the Maxwell color box, furnishing the box with special slits with movable jaws to facilitate the selection of adjacent spectral patches. A possible arrangement of apparatus is shown in Fig. 2. $C$ is the surface whose color is to be measured. $W$ is a standard white surface under the same illumination. At $S_1$ and $S_2$ are adjustable slits, whose design will be discussed below. $L_1$ and $L_2$ are two lenses placed at their focal distances from $S_1$ and $S_2$. $F$ is a compound photometric prism designed to reflect the light from $L_1$ through one part of the field as viewed from $E$, and the light from $L_2$ through another part, the dividing edge being as fine as possible. $P$ is a dispersing prism or grating, $L_3$ the telescope lens, and $E$ the narrow viewing slit. The eye, placed at $E$, sees the photometric surface $F$ in light of the color of the portion of the spectrum received at $E$.

The two slits, $S_1$ and $S_2$, are placed at corresponding positions in the two spectrometers formed by $L_1$ and $L_2$, and are arranged to move laterally across the focal planes of $L_1$ and $L_2$ at the same rate. The slits are made according to a plan utilized by the writer some years ago in studying the summational capacity of the flicker photometer.[5] Each slit consists of a preceding and a following jaw. The two jaws are first placed in contact at a given wave-length, which is the limit to one of the standard spectral patches; then the preceding edge is moved to the other limiting wave-length, and a measurement made. Then the following edge is brought up to contact and the process repeated. By mounting the two corresponding jaws on a single sliding piece the adjustment of both slits may be made simultaneously.

In order to make a photometric match a variable aperture sector $D_1$ is placed before the white light slit (a compensating simple sector $D_2$ on the test color side is necessary to match the transmission of the variable sector at its fullest opening). The process of making a measurement consists in isolating the suc-

[5] "The Addition of Luminosities of Different Color," Phil. Mag., Dec. 1912, p. 845.

cessive monochromatic patches, and for each one varying the sector transmission until photometric match is obtained.

A pictorial plot of such a measurement is shown in Fig. 3. Such a plot constitutes a necessary and sufficient record of any type of color. It is to be noted that this plot may be constructed from an ordinary spectrophotometric series of measurements equally well if the latter is available, and that this plot may be used as a spectrophotometric one, to derive color sensation values.

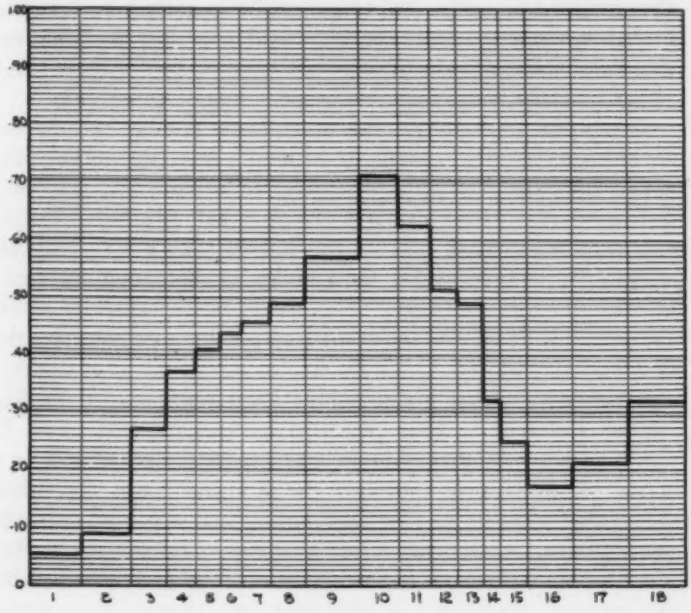



*Fig. 3*

Plot of measurement, consisting of a photometric setting, recorded in per cent, for each monochromatic patch of equal hue scale width.

Some practical questions which would arise in the construction of the hue scale spectrophotometer may be noted. Obviously the spectrum should be as long as possible in order to achieve practically the sharp demarkation of the spectrum patches. The longer the collimators, the greater the actual slit width corre-

sponding to a given wave-length interval. The longer the telescope, the narrower will the eye-slit be compared to the collimator slit, and hence the more nearly will it approach the geometrical line which is called for to avoid overlapping of the patches. Similarly the greater the dispersion of the prism the longer will be the spectrum, and the easier to insure its accurate subdivision. For the full realization of the scheme, an apparatus of at least two meters optical length should be used. The illumination of the test and standard surfaces would have to be as intense as possible. Probably sunlight would be the most practical illumination; although, since any continuous spectrum source will do, high power tungsten lamps might suffice.

The question naturally comes up as to the number and spacing of the monochromatic patches to be used. As to the number, the upper limit for measuring purposes, to avoid color difference settings, is that fixed by each patch being half a least perceptible difference step. With patches this narrow, any color whatever, including isolated spectral lines, could be measured by exact no-color-difference settings. This would mean probably about 250 patches. It may be confidently stated, however, that for the vast majority of colors, far fewer patches would be necessary. The minimum which would permit the recognition of the five minima and four maxima exhibited by the hue scale (Fig. 1) would be nine. The most useful and practical number will be a matter for experiment to decide. It is indeed quite likely that the method could be standardized on the basis of several different patch numbers, each representing a different degree of refinement of measurement, the proper patch number being determined by the spectral complexity of the color and degree of accuracy required. If these numbers were each fractional parts such as $\frac{1}{2}$ or $\frac{1}{4}$ of the most refined scale, they could all be plotted on the same coördinate paper. For instance a 36 patch scale could be used to plot 18 and 9 patch observations.

The spacing of the color patches should be according to the hue sensibility curve for the *average* eye (Fig. 1). This curve, of course, varies from one individual to another, which will mean that with wide patches the approximation to no-color-difference in

the photometric setting will vary. This will, however, only have a second order effect on the result.

Thus far the discussion has been entirely confined to the problem of measuring the color. An equally important feature of a system of colorimetry is means for reproducing the measured color. To do this with the instrument described, the procedure

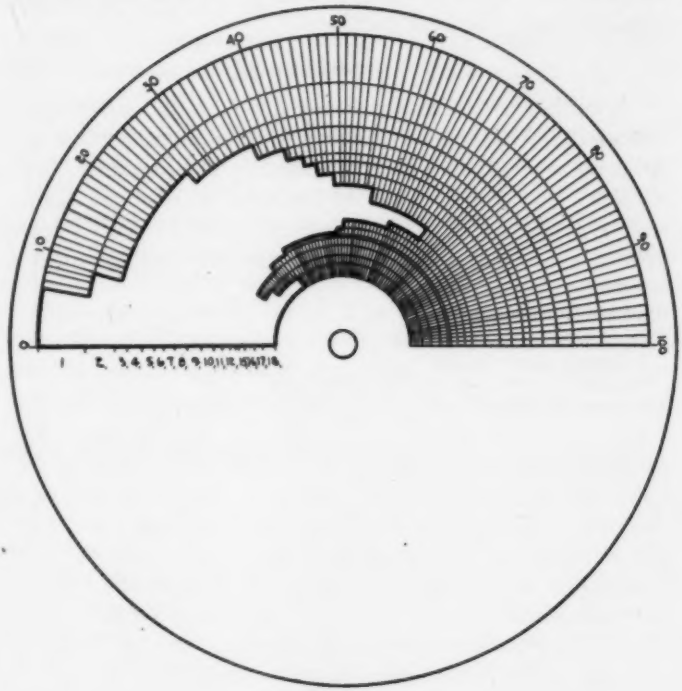


*Fig. 4*

Spectrum template disc for synthesis of color plotted in Fig. 3.

will be to open both slits to their full extent, and to place in the plane of $S_2$ a sector disc (Fig. 4) so cut that its opening at the radial distance corresponding to each wave-length shall be proportional to the ordinate of Figure 3 at that wave length, thus synthesizing the color according to a method due to Abney

already utilized by the writer.[6] Such a set of sector templates will constitute, when used with the appropriate instrument, a set of working color standards exactly specified. To insure interchangeability of these templates, all hue-scale spectrophotometers should be manufactured of standard collimator focus and with identical prisms. If this is done blanks for cutting the template discs could be stamped with patch limit and percentage opening marks ready for cutting.

## SUMMARY

A method of color measurement is suggested which consists essentially of the spectrophotometry of adjacent patches of the spectrum, each patch of a width fixed by the hue scale, and the number of patches dictated by the kind of color and degree of accuracy required. An instrument is described by which the measurements can be made, and in which the color can be reproduced for study.

RESEARCH LABORATORIES,
THE AMERICAN TELEPHONE AND
TELEGRAPH COMPANY, AND
WESTERN ELECTRIC COMPANY, INC., NEW YORK
June 28, 1921.

[6] Ives and Brady, "An Apparatus for the Spectroscopic Synthesis of Color." Jnl. Franklin Institute, July 1914, p. 89.

# A NEW X-RAY DIFFRACTION APPARATUS

BY

WHEELER P. DAVEY

The use of monochromatic X-Rays for the determination of crystal structure is well known. It is the purpose of this article to describe an apparatus by which the characteristic X-Ray diffraction patterns of fifteen powdered crystals may be taken at once. Each pattern is recorded on a strip of photographic film 1⅞ x 16 inches, placed in a cassette which automatically holds the film on the arc of a circle of 8 inch radius, having at its center the glass tube containing the specimen of powdered crystal. These cassettes are placed radially on a circular horizontal table at a fixed distance from a vertical water-cooled Mo-target Coolidge tube. The fifteen specimens thus lie along the edges of a horizontal polygon whose center is on the axis of the Coolidge tubes. The Coolidge tube is excited by a transformer placed under the table. In this way, the whole apparatus, exclusive of the switchboard, occupies a space 3 feet in diameter and about 4½ feet high (see Fig. 1).

The time required to obtain a satisfactory record of the diffraction pattern varies from 5 to 12 hours for substances like NaCl to 48 to 70 hours for substances like CuCl and CsI. The diffraction pattern may be interpreted directly in terms of the interplaner distances in the crystal by means of a metal scale which is calibrated in Ångstrom units. If the crystal happens to belong to the cubic, tetragonal, or hexagonal systems its structure may be interpreted directly from these interplaner distances by means of graphs described below.

## THE TABLE

The table-top consists of a disk of steel ½ inch thick, provided with 15 radial guides for the cassettes. A hole 12 23/64 inches in diameter is cut in the center. In this hole is fitted a steel cylinder 11 9/16 inches high which acts as a support for the slit system, and at the same time, because of the long wave length of X-Rays

employed, provides X-Ray protection for the investigator. Fifteen rectangular openings at the level of the slit system allow the rays to reach the glass tubes containing the specimens. Under each of these openings is another opening which may be used when desired for calibrating the photographic film. It is, of course, imperative, that all openings not covered by cassettes be blocked off by sheet lead or by ⅛ inch sheet brass to avoid danger to the investigator. The lead or brass should overlap the holes by at least half an inch.

*Fig. 1A*—X-Ray Diffraction apparatus (doors closed)

The top of the cylinder is closed by a flat brass cover which also acts as a rigid support for the anode end of the Coolidge tube. The cathode end of the tube hangs freely. The cylinder and cover are machined so that the axis of the tube automatically

coincides with the axis of the cylinder. A set-screw allows a vertical adjustment of the tube along the axis of the cylinder.

The under part of the table is enclosed by a cylinder of heavy sheet iron, reënforced with angle iron, and provided with doors to give access to the high-tension transformer which sits inside (see Fig. 1 B). The primary circuit of the transformer is carried

*Fig. 1B.* X-Ray Diffraction apparatus (doors open)

across the front of these doors in such a way that the doors can not be opened without opening the primary circuit (see Fig. 1 A). In this way, when the X-Ray tube is in operation, the operator is protected from accidental contact with the high potential.

The whole table is mounted on heavy castors so that it may be easily moved from place to place.

## THE TUBE AND ITS ACCESSORIES

The X-Ray tube is of the usual Coolidge type, but with a special water-cooled anode. This anode consists of a hollow copper rod at the end of which is fastened a molybdenum button. The shape of the rod is such as to provide an easy heat-path from the button to a large surface of water, so that local boiling is avoided. The face of the Mo button is perpendicular to the axis of the tube, so that X-rays may be taken off all the way around the anode.

The cathode is the standard 30 ma. cathode commonly found in the Radiator Type Coolidge tubes supplied to roentgenologists. Because of the large anode-cathode distance and the low voltage employed (30 $kv_{RMS}$) the tube shows space-charge characteristics, that is, the voltage drop immediately in front of the cathode filament is so low that when the filament operates at normal heating current (say 4.5—4.75 amperes), fewer electrons are dragged from the cathode to the anode than are evaporated from the filament. For this reason, at a definite voltage across the tube the current is independent of the filament temperature provided that the temperature exceeds a certain minimal value. A tube operated so as to show space—charge characteristics appears to be more efficient in crystal analysis work than one having the characteristics of the ordinary Coolidge tube, in which the space-charge characteristics are noticeable at lower voltages, but in which a higher percentage of the electrons is utilized at 30 kv. There is, besides, this additional advantage, that the tube load is nearly constant even though the filament is heated from a variable source of potential, provided that the variation in voltage on the filament circuit is less than 5%.

The maximum allowable filament current for 1000 hours life is 4.75 amperes. The maximum tube voltage for efficient production of Mo characteristic rays is 30 $kv_{RMS}$. At this voltage the tube current should be about 30 ma, but will vary from 28 to 35 ma from one tube to another.

Dimensions of the tube are shown in Fig. 2.

The radiation from the tube is filtered through two filters of $ZrO_2$, each having .05 g. of $ZrO_2$ per $cm^2$. One filter is built into

the slit system,—the other into the cassettes. The first filter eliminates most of the "white" (general) radiation, and a large fraction of the Mo $\beta$ doublet (.63 Å). The second filter makes the rest of the Mo $\beta$ doublet negligible, and eliminates a large part of such characteristic rays as may be given off by the specimen itself. The diffraction pattern, as recorded on the photographic film, is therefore caused by the Mo $\alpha$ doublet (average wave length .712 Å).

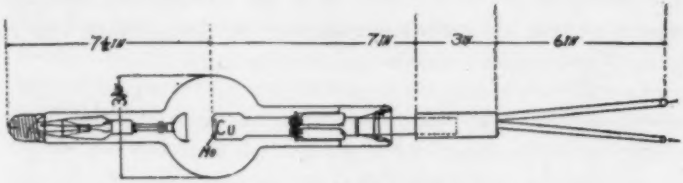



*Fig. 2.* Diagram of water cooled Coolidge tube

The optimum D. C. voltage for the production of Mo $\alpha$ is 28–30 kv,[1] but operation on A. C. at 30 kv $_{MAX}$ makes the time required for a satisfactory film so long that it is prohibitive for ordinary work. Experience shows that 30 kv $_{RMS}$ is about the maximum voltage at which the "white" radiation may be easily filtered from the Mo characteristic rays. For this reason it was adopted as the standard operating voltage for this apparatus.

The Coolidge tube is operated directly from a transformer without any additional rectifying device. This necessitates a special design of transformer, for, although the secondary must supply power only on one half of the wave, the voltage of the inverse half-wave must not greatly exceed that of the useful half-wave. Otherwise there will be considerable trouble from corona during the inverse half-wave. The transformer has only one developed terminal, the other end of the secondary being grounded through the milliammeter. A protective spark-gap is provided across the milliammeter circuit so that the meter is always safe to handle even if one of its connections is broken. The filament of the Coolidge tube is heated by an extra coil at

[1] A. W. Hull, "A New Method of Crystal Analysis," *Phys. Rev.* 10, p. 661, 1917.

the high-potential end of the secondary. Both ends of this coil, as well as the high-potential end of the secondary, are brought out through the high-potential terminal to a Kearsley stabilizer,[2] which automatically lowers the filament temperature in case of excessive discharge currents through the tube. Fig. 3 gives a diagram of the connections. From the Kearsley stabilizer, the high-potential filament circuit goes directly to the cathode of the Coolidge tube which hangs directly above it. The anode end of the tube is in electrical contact with the table top, as described above, so that the circuit is completed by the grounded metal

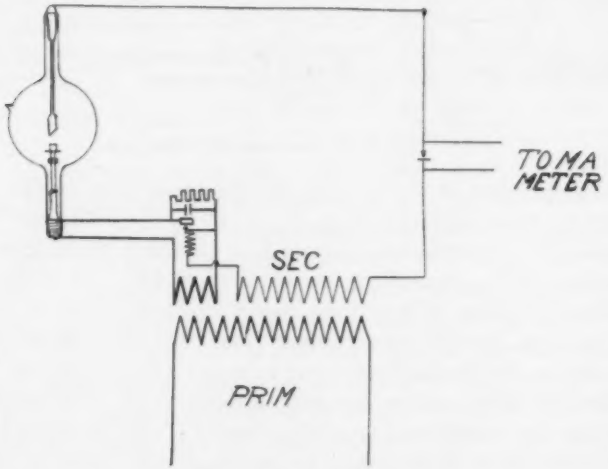



*Fig. 3.* Wiring diagram

table, and the milliammeter back to the transformer. In this way the high potential is entirely surrounded by grounded metal.

The switchboard contains the customary circuit breakers, operating switch, an auto-transformer for adjusting the primary voltage of the transformer, a voltmeter for reading the primary voltage, a milliammeter and a pressure-stat. This pressure-stat is connected to the outlet end of the water cooled anode of

[2] W. J. Kearsley, Jr., "A New Type of Stabilizer for Use with the Coolidge Tube," *Journal of Radiology*, July, 1921.

the Coolidge tube. In case the water pressure falls too low, the pressure-stat short-circuits the power line through a small resistance, thus tripping the circuit breakers and protecting the tube from being burned out. The water pressure at the inlet end of the tube should be not less than 25 nor more than 80 lb. per square inch. In setting up the apparatus, enough resistance to flow of water at the outlet is provided so that the pressure-stat opens the protective short-circuit when the inlet pressure is 25 lb. per square inch.

A small centrifugal blower, mounted alongside the transformer, provides an air blast against the cathode neck of the Coolidge tube. This prevents local heating of the glass by radiation from the cathode during continuous operation, which might otherwise evolve gas and thus destroy the space-charge characteristics of the tube, with consequent loss of the tube due to overload.

## The Slit System

The slit system is suspended from a disk of ¼-inch plate glass which has a hole 6¼ inches in diameter cut in the center. The system is composed of 15 radial sets of 3 slits each,—the first two of which define the beam, while the third cuts off most of the secondary rays produced by the edges of the second slit. The whole slit system is formed on the surface of a cone whose basal angle is 5°. In this way each of the 15 slits utilizes the full projection of the focal spot of the Coolidge tube. The first slits are separated from each other by small brass partitions hung from the glass disk. The upper edges of these slits are formed by the edge of the glass disk itself. The lower edges are formed by the edges of small sectors of glass suspended between these partitions and the second slit. In this way the first slit is completely insulated from ground so as to avoid corona between it and the bulb of the Coolidge tube with consequent loss of the tube. If the first slit had not been insulated, it would have been necessary to have placed it further from the wall of the Coolidge tube, or to have inserted sheets of insulation (such as insulating paper or fiber) between the slit and the tube. In either case the time required to photograph the diffraction pattern would have been increased.

The second and third slits are composed entirely of brass, which seems to be about as good as lead for the wave length employed. They are supported from the glass disk, but are electrically connected to the grounded cylinder of the table top by small set screws which prevent displacement of the slit system. The slit system is shown in Fig. 4.

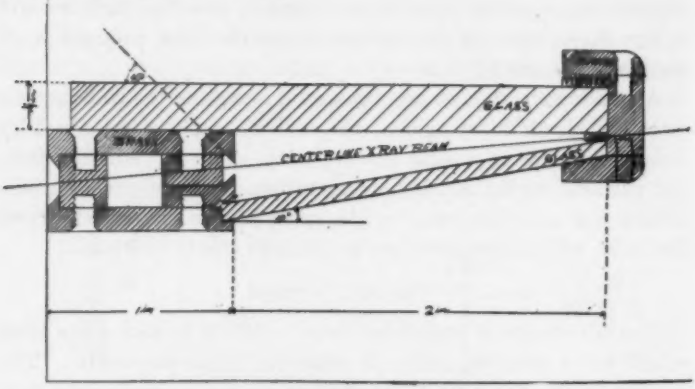



*Fig. 4.* Diagram of slit system

One of the filters mentioned above is fastened to the inner ring of slits, but is not shown in the diagram. Except for 15 rectangular spaces immediately in front of the slits, the whole surface of the filter is protected by nickel-plated brass from deterioration due to heat from the cathode reflected from the focal spot. At the slits, the white cardboard on which the filter is mounted is used for the same purpose.

The writer is well aware that numerous special types of slit systems have been devised by means of which X-Ray diffraction patterns may be photographed in less time than with the system described above. Although their merits for some highly specialized investigations are undoubted, they are hardly suited for general crystal research, nor for apparatus intended for most forms of routine factory-control work, either because of the large quantity of specimen required, or the cost of preparing the specimen in some special shape, or because certain lines in the pattern are enhanced at the expense of others.

## THE CASSETTES, FILMS, AND SPECIMENS

The cassettes (Fig. 5) serve not only as holders for the films, but also as holders for the specimens. The powdered specimens (preferably 200 mesh) may be mixed with pyroxylin and formed into thin, flat sheets, or they may be packed into thin-walled tubes of about 1/32 inch inside diameter, of paper, celluloid or special glass. The glass tubes are preferred for most substances because of the possibility of sealing the specimens so that they can not be affected by oxygen or moisture. Whether the specimen is prepared in the form of a flat sheet or packed in a tube, the specimen-holder on the cassette automatically holds it in the path of the

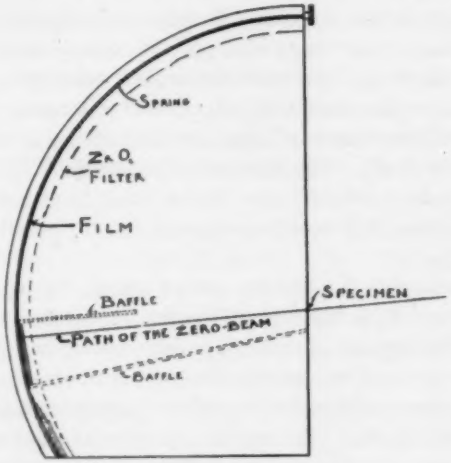



*Fig. 5.* Cassette

X-Ray beam. In case the specimen contains elements of high atomic weight, it should be mixed with corn-starch or other amorphous substance, to decrease the capacity of the total mass.

A strip of thin, black celluloid is fastened to the cassette in the form of an arc, 8 inches in radius, with the specimen at the center. The photographic film is held against this by a wide flat brass spring which is drawn up tight by a screw. A light-

baffle over the spring prevents the film from becoming light-struck. The most satisfactory film for X-Ray diffraction work seems to be the Eastman "Dupli-tized X-Ray" film.

A "staircase" of copper is placed as an absorber in the path of the zero-beam. In this way there is always some part of the zero line on the film which has an exposure comparable to that of the first lines on the diffraction pattern. A molybdenum staircase is provided below the zero-beam for calibrating the photographic film.

A septum in the median plane of the cassette divides it into two symmetrical chambers, so that diffraction patterns of two substances may be taken on the same film for purposes of comparison. This is done by filling the glass specimen-tube half full of one substance, inserting a tiny plug of cotton, and then filling the remainder of the tube with the second substance. In work requiring the measurement of inter-planer distances with great accuracy it is recommended that one-half of the specimen-tube be filled with NaCl. The theoretical spacings of NaCl (side of elementary cube = 2.810Å) may then be used to give a correction curve for the film, thus avoiding errors due to any possible changes in the film itself.

A "range finder" is provided for adjusting the height of the Coolidge tube. It is built to fit the guides on the table top so that it may be slipped into place opposite any one of the 15 slits. A tungsten wire acts as the "specimen." A fluorescent screen is mounted at the outside end in a position corresponding to that of the film in the cassette. The screen is faced with glass as a partial X-Ray protection to the operator. The Coolidge tube is at the proper height when the fluorescent screen shows that the shadow of the tungsten wire is in the center of the zero-line.

## THE DIFFRACTION-PATTERN

The photograph of the X-Ray diffraction pattern looks much like the familiar picture of a line spectrum. The diffraction pattern of NaCl, taken with this apparatus, is shown in Fig. 6. A full account of the mathematical theory of the pattern is given elsewhere by A. W. Hull.[3]

[3] A. W. Hull, loc. cit.

Using the classical equation for diffraction,

$$2d = n\lambda \sin \theta$$

a scale has been calculated by which each line on the film may be directly interpreted in terms of the distance between the planes of atoms which produced the line. The apparatus has thus made a record of the diffraction pattern of the specimen and has enabled

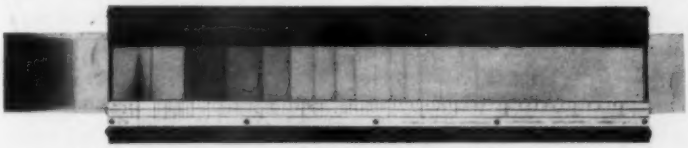


*Fig. 6.* Diffraction pattern of NaCl on scale, ready for measurement

the experimenter to record in Ångstrom units the spacings of all planes in the crystal which are more than .55 Å apart. Theoretically a diffraction pattern should be given by all planes having a spacing greater than $\lambda/2$ (= .356 Å), but except with very long exposure-time the lines from planes less than .55 Å are so faint as to be useless. Indeed, where a solution of the crystal structure can be found at all, it is usually possible to make a valid solution using only lines corresponding to planer distances of more than .80 Å. When the doublet is resolved ($\lambda$ = .710 Å, $\lambda_2$ = .714 Å) the readings of the two lines as given by the scale will be in the ratio of 1 : 1.005. In such a case the pattern is to be considered as showing a single line situated half-way between the two actual lines. The order of accuracy of readings of lines by means of the scale increases from the left hand end of the scale (large values) to the right hand end (small values). At 1.00 Å it is of the order of 0.1 per cent. The limiting feature is the accuracy of the investigator in estimating the center of the line on the film. In case a solution of the crystal structure is found by the graphical method mentioned below, the final result is usually accurate to about ¼ per cent. It can then be checked by numerical calculation to about 0.2 per cent.

## USES

The apparatus described above has two fields of usefulness,—(1) in the identification of materials, (2) in the determination of crystal structure.

(1) All solid metals and their alloys, and most solid compounds, are crystalline and show X-Ray diffraction patterns. Waxes at ordinary temperature, and glasses are amorphous and show no diffraction patterns. The diffraction pattern of a given substance, as measured quantitatively on the scale described above, is as characteristic of that substance as the density, solubility, melting-point or ability to form chemical compounds. As far as is now known, no two substances have identically the same diffraction pattern. At first it was thought that the patterns of Mo and W were identical, but Hull has since shown[4] that the measurements of their diffraction patterns differ by about ¼ per cent. All other substances so far investigated which happen to have diffraction patterns of similar appearance are found, upon measurement with the scale described above, to differ from each other by several per cent. The apparatus therefore offers a convenient method for the qualitative analysis of crystaline substances. Any substance is completely identified when its pattern, measured on the scale, exactly matches the pattern of some substance whose identity is known. A large mass of such data is already available and will be published shortly in compact form.

This method of analysis has three advantages,—(a) only .001 cc. of the specimen (ground to 200 mesh) is required; (b) the original sample is still available after the analysis is completed; (c) in a mixture of two compounds, the *state of combination* may be determined[5]—i.e.,

NaF+KCl can be distinguished from
NaCl+KF

The ease with which substances may be identified should make the method useful as a means of laboratory control of factory products, especially in the case of metals and their alloys. The method of crystal analysis offers the most accurate method known for the determination of coefficient of expansion and density. In both cases a solution of the structure of the crystal is necessary, as described below.

[4] A. W. Hull. *Physical Review*, 17, p. 5, 1921.

[5] A. W. Hull, "A New Method of Chemical Analysis," *Jour. Am. Chem. Soc.* 41, p. 1168, 1919.

(2) It is often desirable, not only in purely scientific investigations, but also in factory-control work, to determine the arrangement of atoms in the crystal. In case the crystal happens to belong to the cubic, hexagonal, or tetragonal systems (and most metals, alloys, halogen compounds, nitrates of monovalent metals, and carbonates of bivalent metals do), the crystal structure may be solved directly by the following method.[6]

The edge of a strip of paper is laid along the logarithmic scale of abscissae which is found at the bottom of each of the plots of Fig. 7, and the distance between planes in the crystal, as read off from the film in Ångstrom units, is plotted on this edge. The strip of paper is now moved across the various plots keeping the edge always parallel to the axes of abscissae. When an exact match is found between the plot and the pattern marked on the edge of the paper then the crystal belongs to the system and subdivision marked on the plot, and the correct axial ratio is given by the intersection of the edge of the paper with the axis of ordinates. It is necessary that every line in the experimental pattern on the strip of paper be represented in the plot. If there is a single experimental line left over, the solution is not valid, no matter how good a match is obtained with the rest of the lines. The only exception to this is when several lines left over in this way can be shown to all fit some other portion of one of the plots, thus indicating that two crystal forms are present. If the plot predicts lines which are not found in the experimental pattern it is necessary to show either that those lines ought, theoretically, to be too faint to be seen on the film, or that the crystal structure is more complicated than that for which the plot was made, so that certain lines disappear by interference. (Thus any pattern given on the triangular close-packing and rhombohedral plots will fit certain lines of the simple triangular plots at the same axial ratio, and all the patterns of the body-centered and face-centered tetragonal structures will similarly fit portions of the simple tetragonal plots.)

[6] For a more complete description of the method see *Hull and Davey*, "Graphical Determination of Hexagonal and Tetragonal Crystal Structures from X-Ray Data," *Physical Review*, 17, p. 5, 1921.

Plots of the cubic system will be found at an axial ratio of 1.00 in the plots of the tetragonal system.[7] The diamond division of the cubic system may be found by omitting the 2nd, 5th, 8th, 13th, and 19th lines, counting from the right hand side of the plot, from the plot of the face-centered division of the cubic system.

After an exact match is found for the experimental pattern, it should be verified by computing the density of the specimen in the following manner,—

(a) If the match is in the cubic system, the density of the substance is given by

$$\rho = n\frac{M\times 1.649\times 10^{-24}}{(d\times 10^{-8})^3}$$

where

$\rho$ is the density

M is the molecular weight of the substance; $1.649\times 10^{-24}$ is the mass in grams of one unit of molecular (or atomic) weight;

d is the side of the elementary cube,—(it is the distance in Ångstroms between the 1 0 0 planes in the crystal);

n is the number of points associated with a unit cube in the crystal lattice. n is
1 for the simple cubic lattice
2 for the body-centered cubic lattice
4 for the face-centered cubic lattice
8 for the diamond-cubic lattice.

If the line corresponding to the "100 planes" is absent (as in the case of the body-centered and diamond divisions) then twice the distance corresponding to the second-order line, called in the plots "100(2)—100," must be used as the side of the elementary cube.

(b) If the match is found in the tetragonal system, the density is obviously

$$\rho = \frac{M\times 1.649\times 10^{-24}}{c(d\times 10^{-8})^3}$$

where $c$ is the axial ratio, and $n$ has the same values as before.

(c) If the match is found in the hexagonal system, find the distance corresponding to the "10·0 planes." (If the match is

[7] They will also be found in the plots of the rhombohedral division of the hexagonal system.

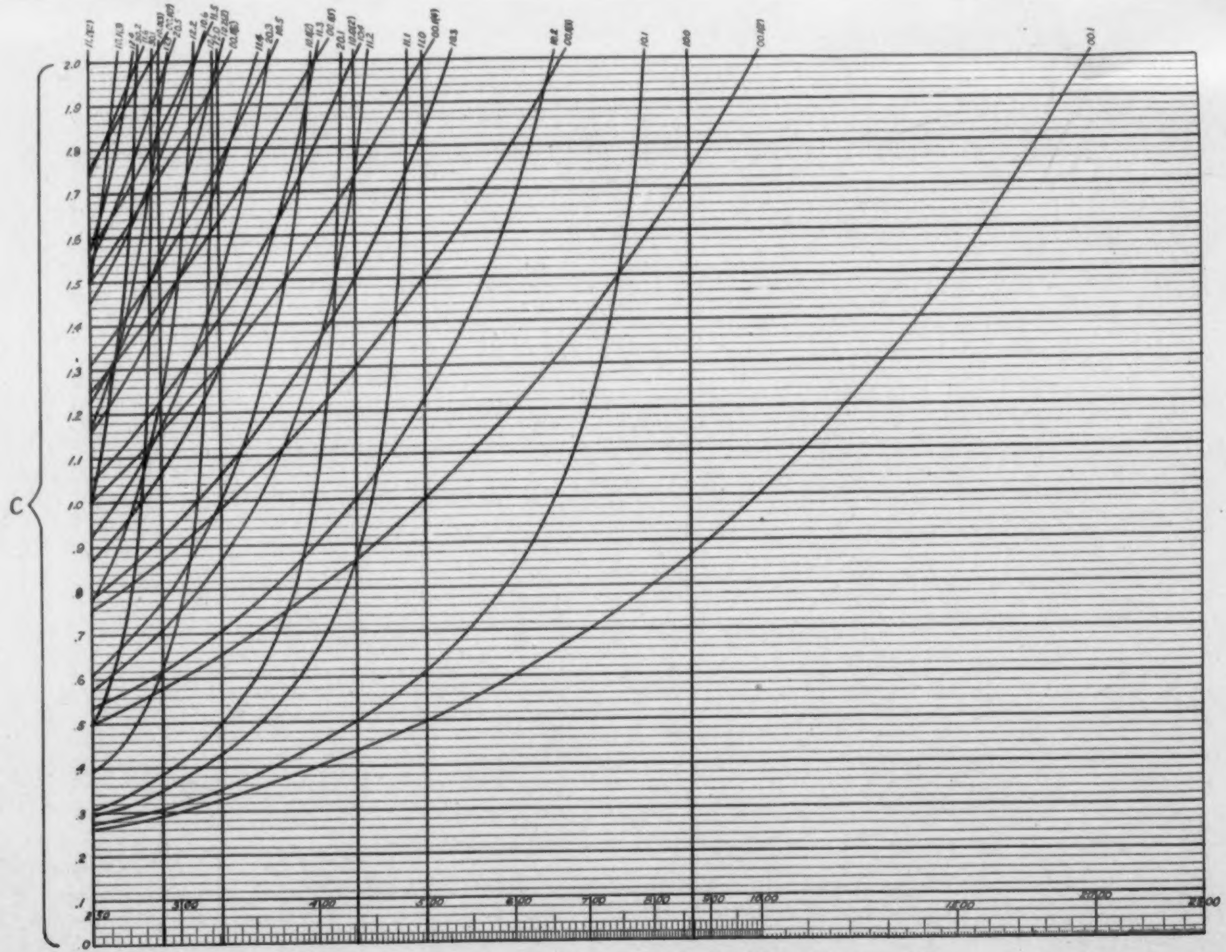



FIG. 7A SIMPLE TRIANGULAR LATTICE (HEXAGONAL SYSTEM)

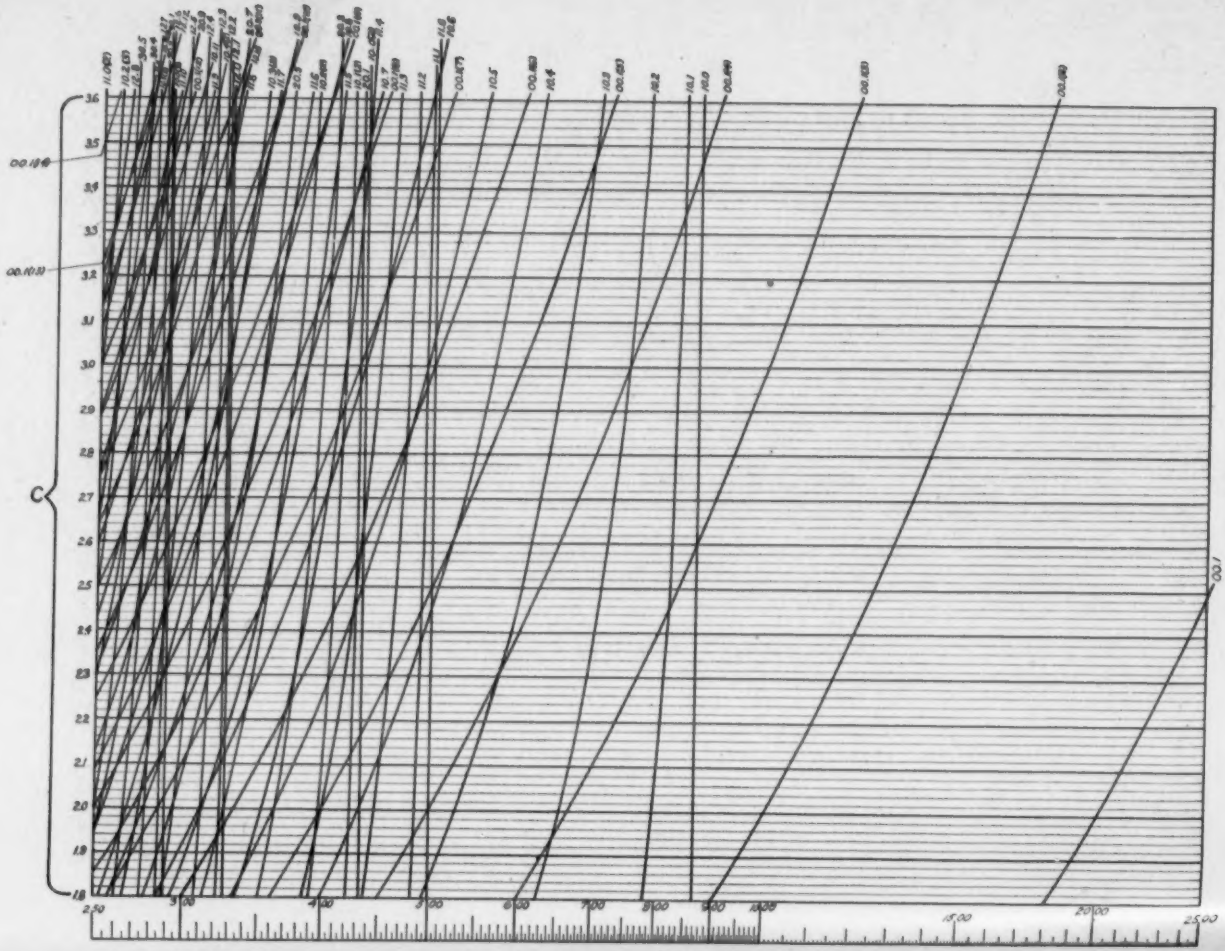



Fig. 7B SIMPLE TRIANGULAR LATTICE (HEXAGONAL SYSTEM)

FIG. 7B SIMPLE TRIANGULAR LATTICE (HEXAGONAL SYSTEM)

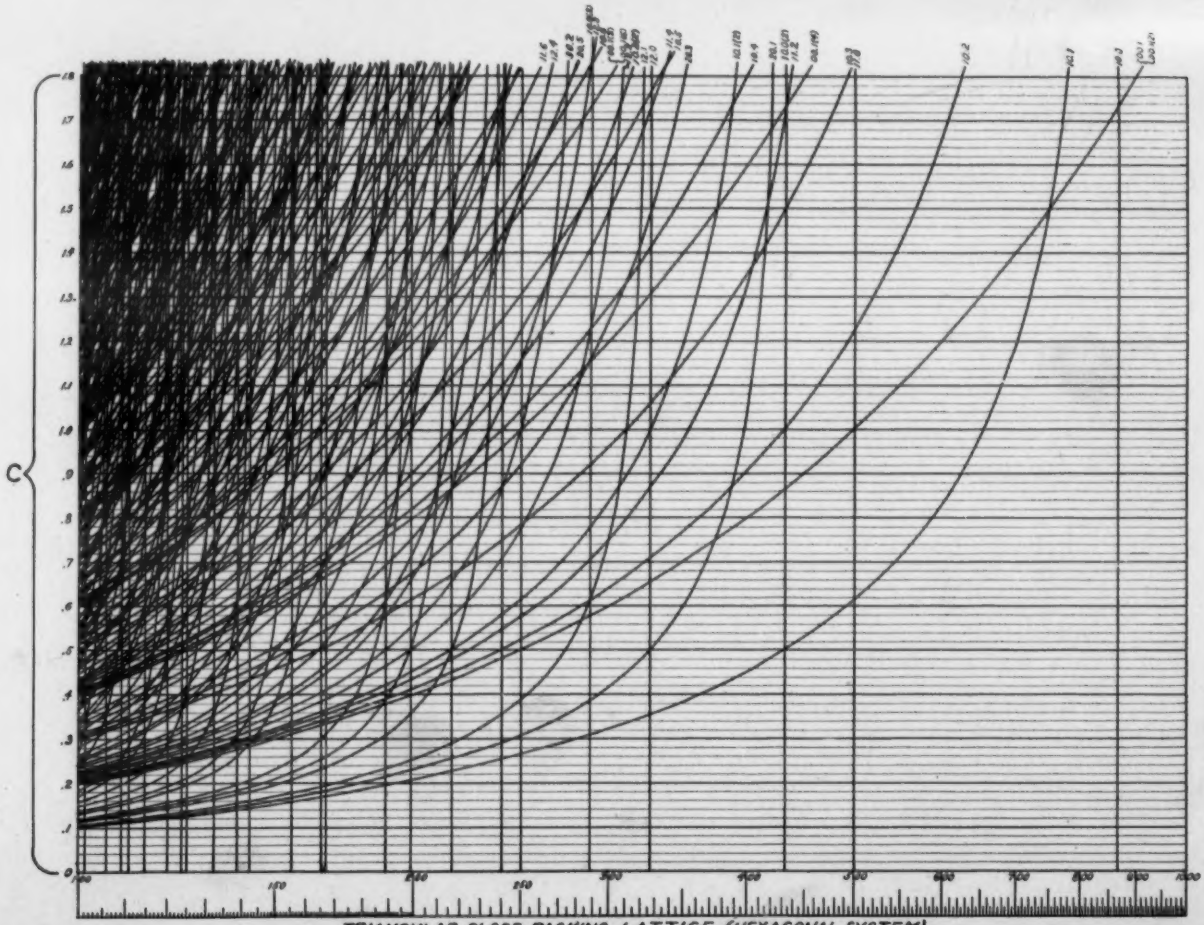



FIG. 7C

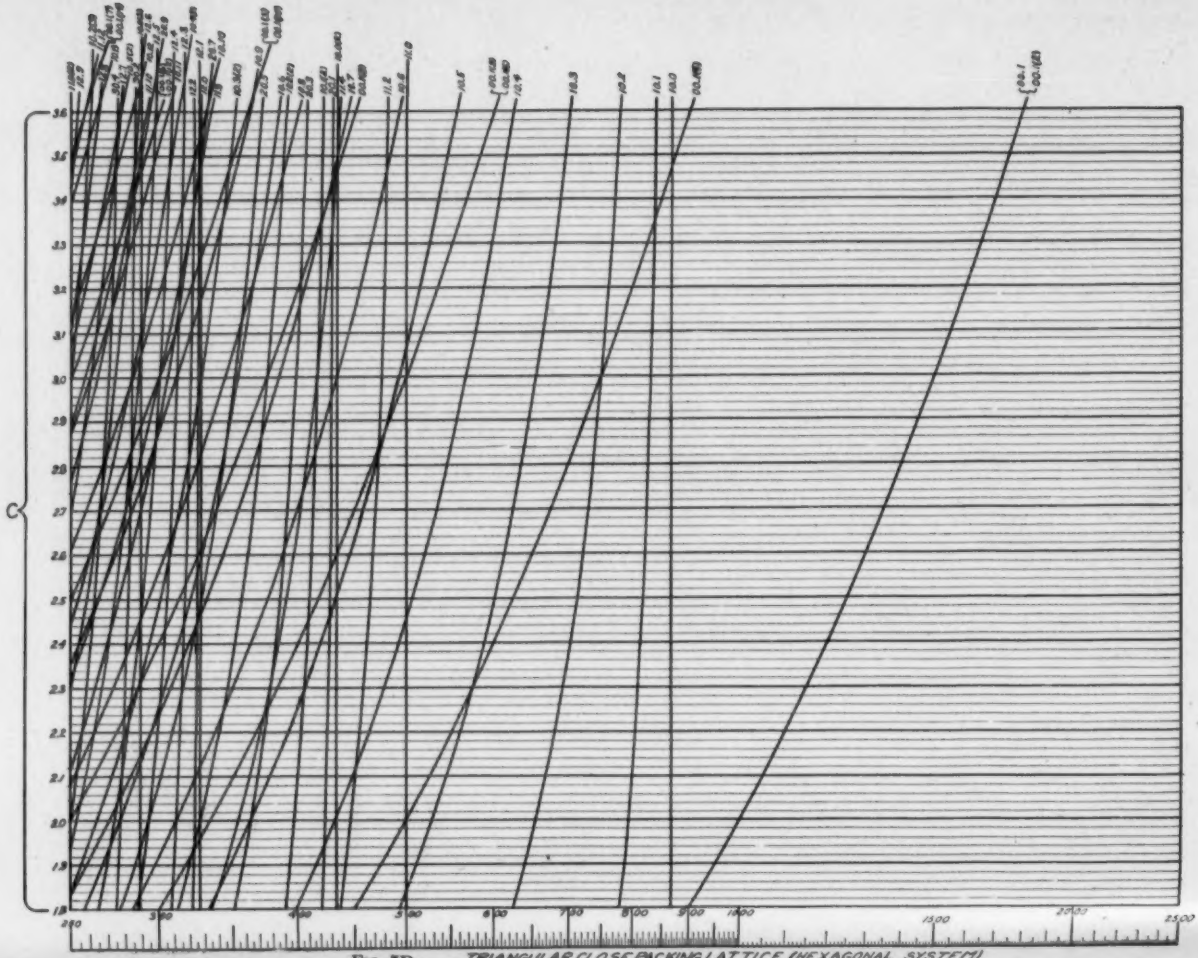



FIG. 7D TRIANGULAR CLOSE PACKING LATTICE (HEXAGONAL SYSTEM)

FIG. 7D TRIANGULAR CLOSE PACKING LATTICE (HEXAGONAL SYSTEM)

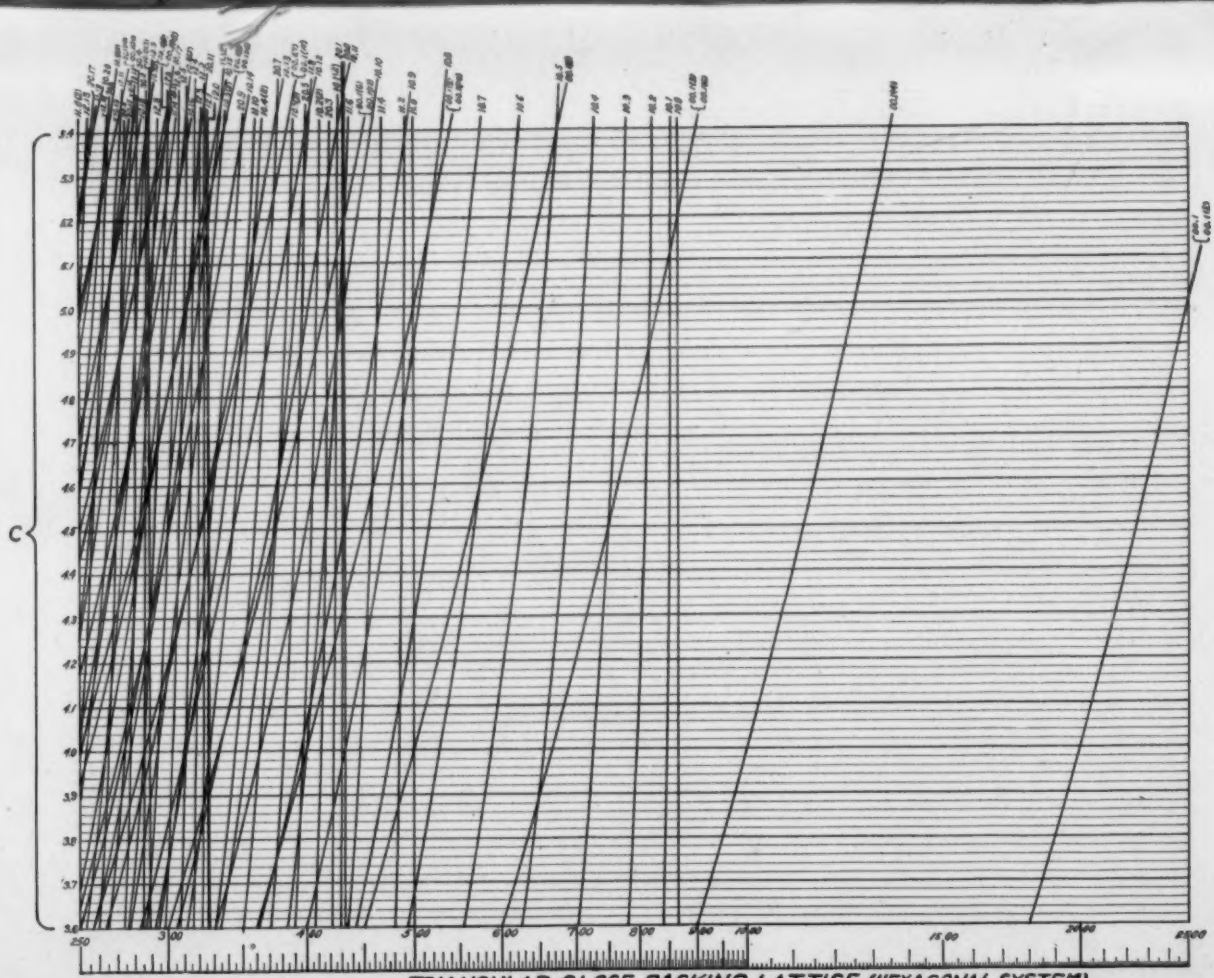



FIG. 7E

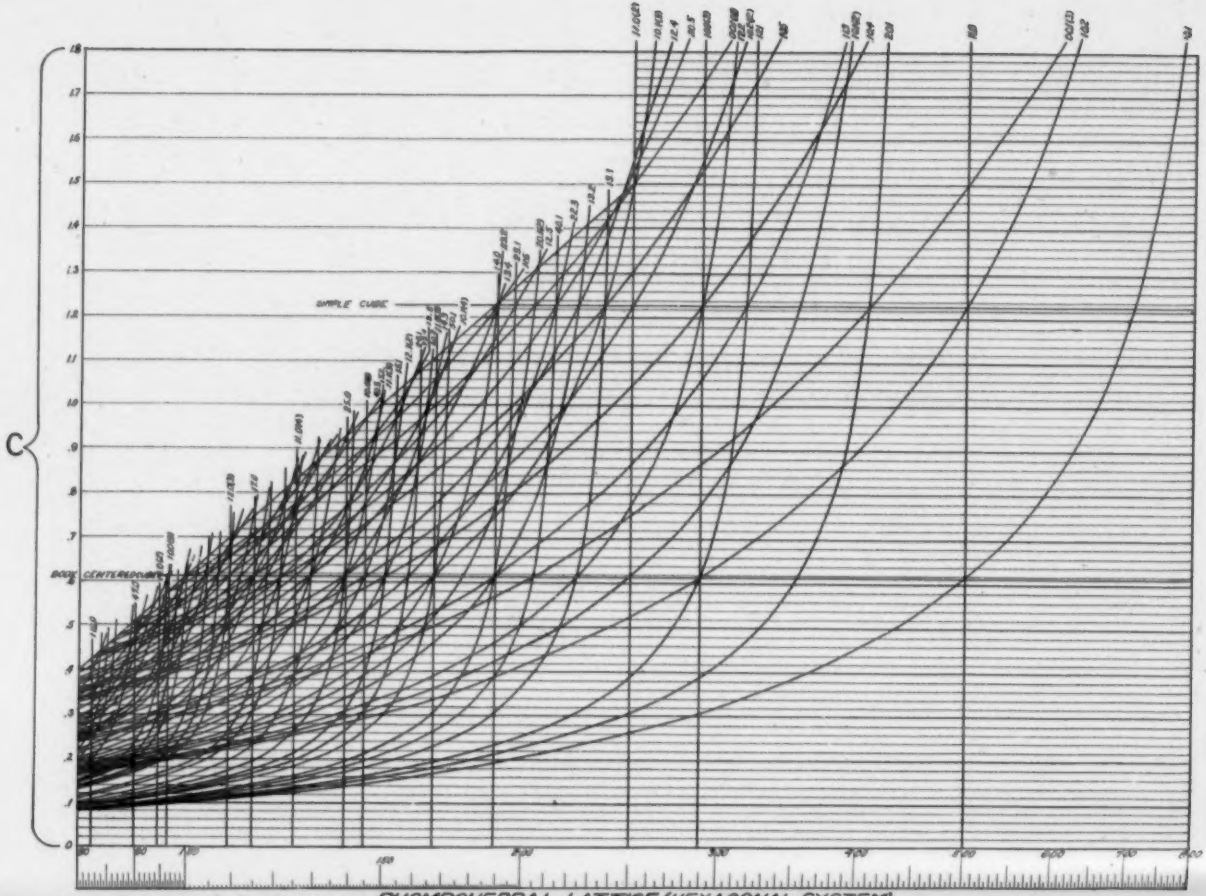



FIG. 7F RHOMBOHEDRAL LATTICE (HEXAGONAL SYSTEM)

FIG. 7F RHOMBOHEDRAL LATTICE (HEXAGONAL SYSTEM)

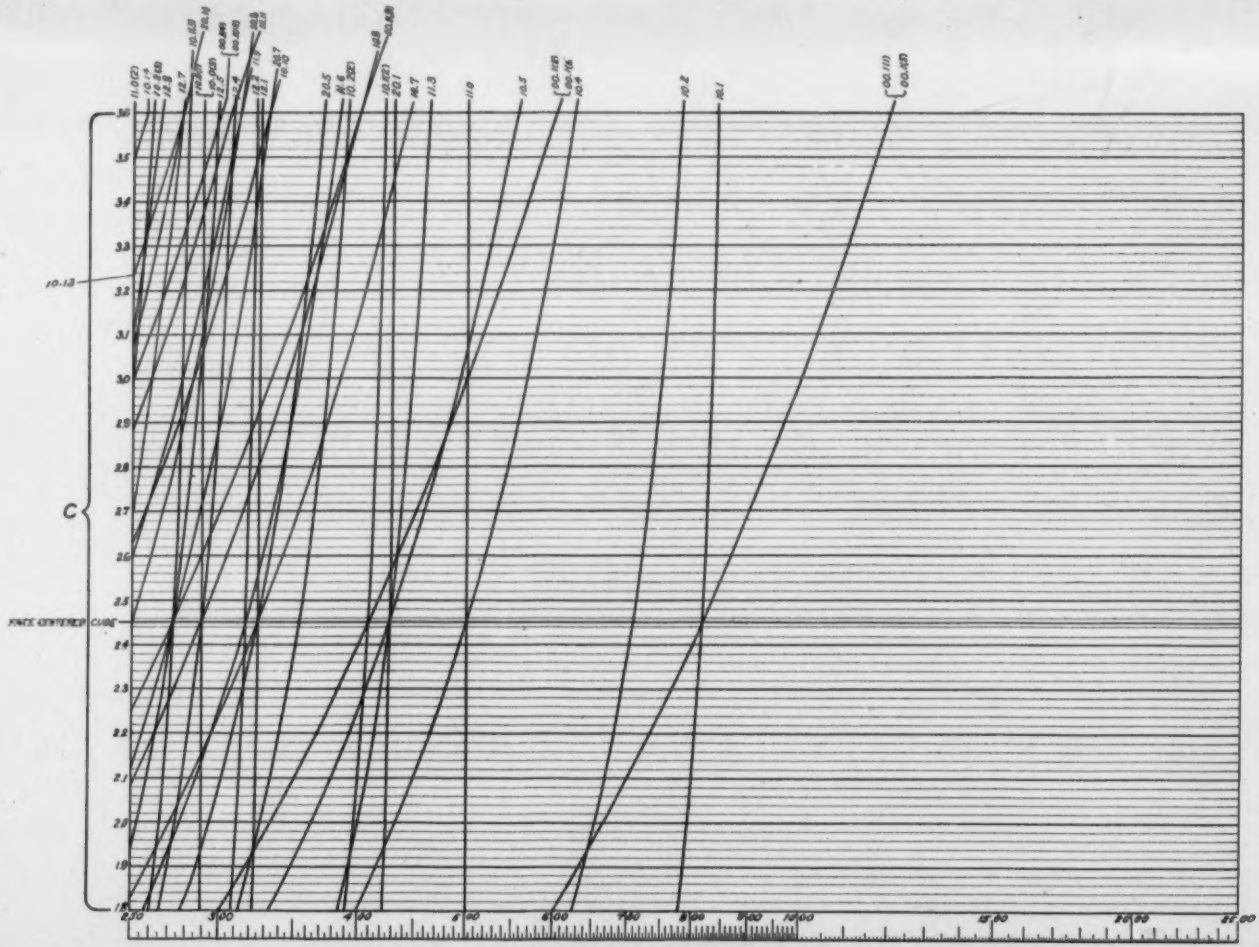



FIG. 7G RHOMBOHEDRAL LATTICE (HEXAGONAL SYSTEM)

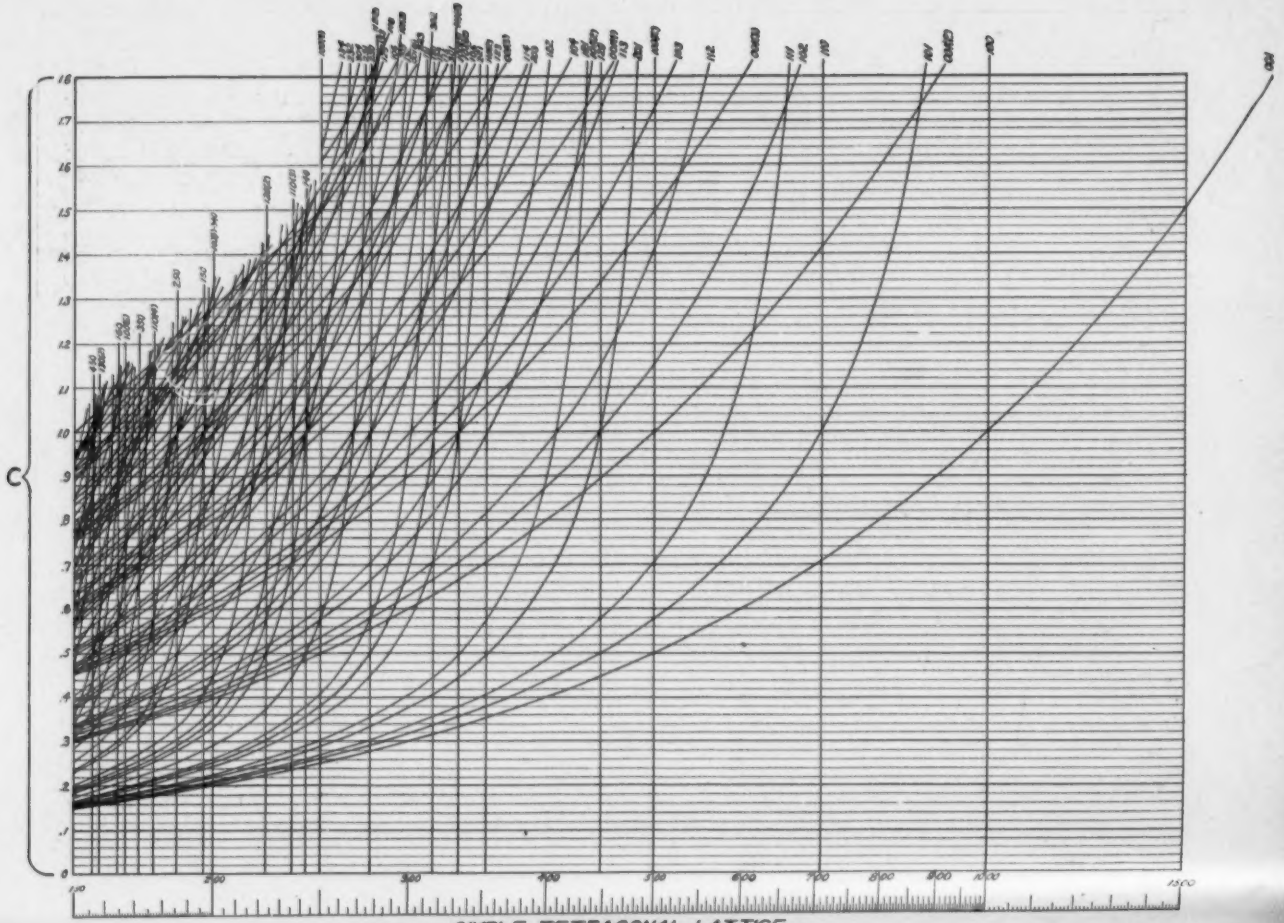
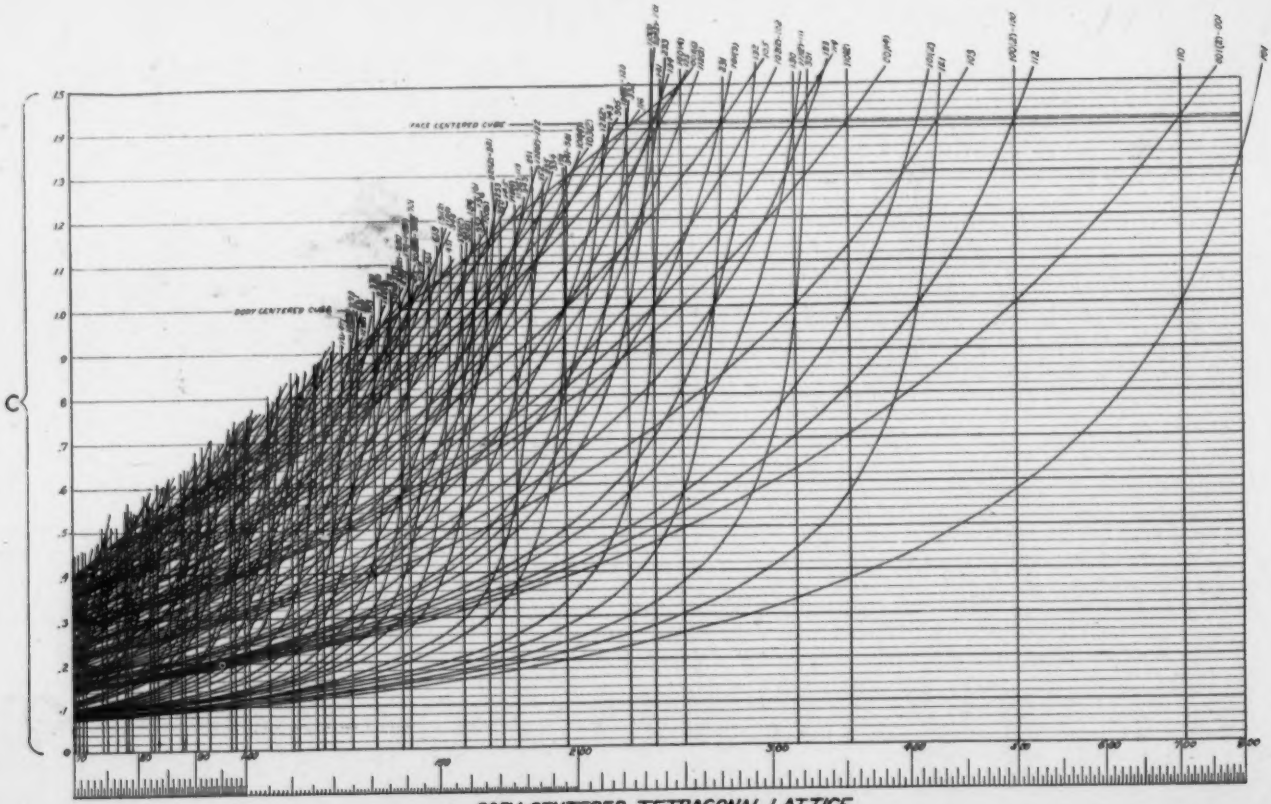



Fig. 7H SIMPLE TETRAGONAL LATTICE

FIG. 7H SIMPLE TETRAGONAL LATTICE




BODY CENTERED TETRAGONAL LATTICE

FIG. 7I

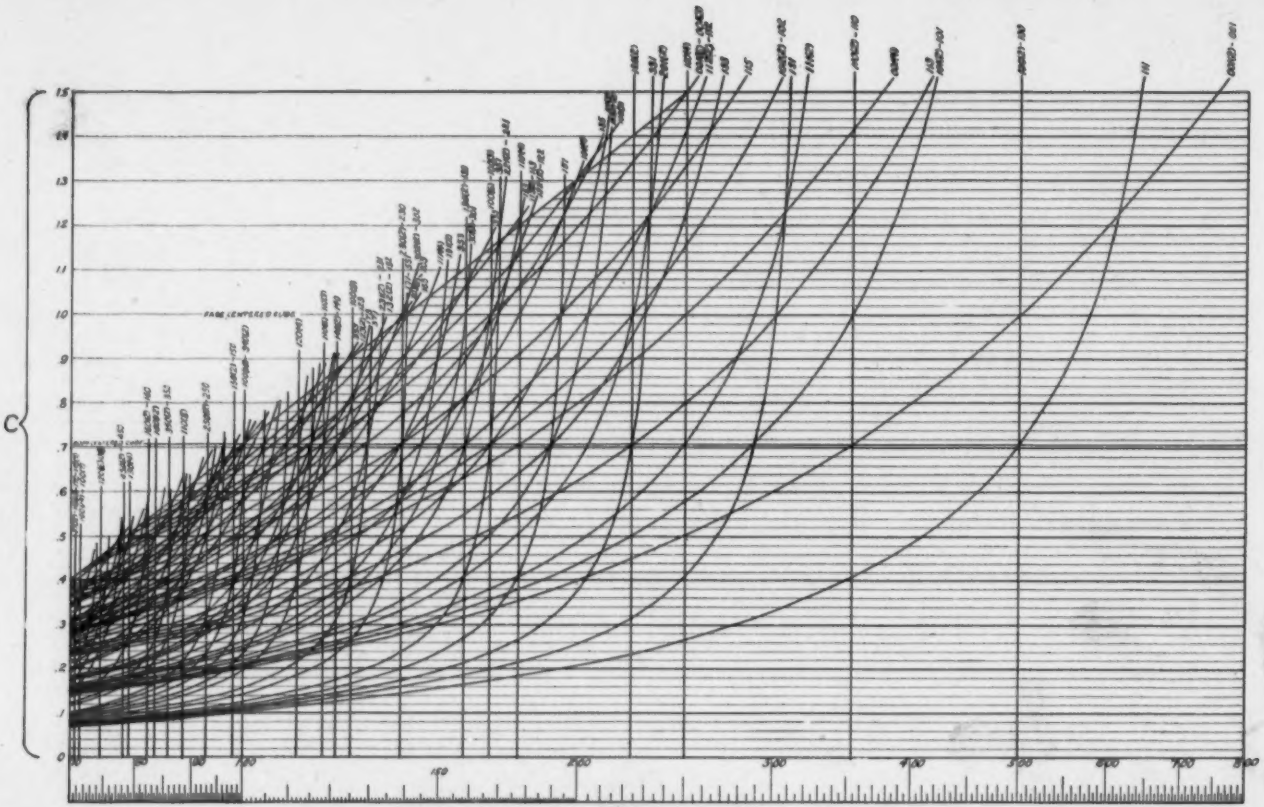



FIG. 71 FACE CENTERED TETRAGONAL LATTICE

found in the rhombohedral division, three times the distance corresponding to the third-order line, called in the plots $\begin{cases}10.0(1)\\10.0(3)\end{cases}$ must be used.) This is the altitude of the equilateral triangle which forms the base of the unit prism. The side of the unit triangle is $2/\sqrt{3}$ times this distance. The density of the specimen is therefore

$$\rho = n\frac{M \times 1.649 \times 10^{-24}}{\sqrt{\tfrac{3}{4}}c\,(a \times 10^{-8})^3}$$

where $a$ is the side of the unit triangle.

n is ½ for a simple triangular lattice
1 for a triangular close-packed lattice
1½ for a rhombohedral lattice.

It is a pleasure to acknowledge the suggestions of A. W. Hull during the course of this work; also the assistance of E. O. Hoffman and H. A. Smith in the design of the slit system and the development of the water-cooled tube; R. Hergenrother in the laborious calculation and drawing of the plots; and Wm. F. Winter in the design and construction of the cassettes and the construction of the slit system.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.

# 1920 REPORT OF COMMITTEE ON PYROMETRY

BY

W. E. FORSYTHE

The following Report for 1920 has been formed by the Chairman following the suggestions and criticisms of members of the Committee especially C. E. Mendenhall. This Report will be divided into three parts:

I. The first part embodies some suggestions for nomenclature and standards.

II. The second part embodies a discussion of ways and means of using an optical pyrometer.

III. The third part embodies a review of the literature on pyrometry for the year.

## I. NOMENCLATURE AND STANDARDS

The most important single item in pyrometry and radiation is temperature. No attempt will be made to define temperature at this point since this idea is pretty well understood and very seldom do we find in the literature that there is any mistake in its use.

Temperature (T), true temperature or shortened to temperature, is to be used when the ordinary idea of temperature is meant. Care always should be taken to specify the scale used. This is a point that is often neglected both by engineers and physicists. Sometimes it is even hard to tell from the context what scale has been used.

The temperatures are to be measured on the International temperature scale. This scale for all practical purposes is the thermodynamic scale of Kelvin. A table will be found in last year's report[1] giving a number of standard temperatures.

*Apparent Temperatures.* There are several methods of measuring temperatures based upon the assumption that the body being studied radiates like a black body. Since there is no known substance that radiates like a black body, numbers so obtained are always different from the true temperature. These different temperatures were at first all referred to as "black-body tempera-

[1] JOUR. OF OPTICAL SOCIETY, *4*, No. 5, p. 310, 1920.

tures," but since there are now at least five such "temperatures," some more definite nomenclature is necessary.

The first of these temperatures is the one that is obtained when measuring non-black bodies with an optical pyrometer calibrated to give the correct temperature of a black body. This temperature is always less than the true temperature and depends upon a measure of the brightness for a particular wave-length interval. It is, in general, different for different wave-length intervals, thus the wave-length to which this temperature corresponds should be specified. Since this temperature is determined from a brightness measure it has been called the *brightness temperature*. The symbol for this is $S_\lambda$, where the subscript ($\lambda$) is to show the wave-length at which the brightness has been measured.

Another useful temperature is one that is determined from a measurement of the total light that is radiated from the source being investigated, as though the source radiated like a black body. This temperature should be known as total brightness temperature and should be designated by $S_t$.

The next temperature of interest is the one that is obtained from the color of the radiated light. Not only is the brightness of a substance for any particular wave-length interval different from that of a black body, but in general the emissive powers for the non-black body will vary from wave-length to wave-length throughout the spectrum. It has been found experimentally that practically all metals when hot radiate in such a manner that the color of the light can be matched by that from a black body at some particular temperature, which temperature has been called the color temperature of the hot metal. Since the light from all radiating substances cannot be accurately matched in color with that of a black body at a particular temperature, it is much more accurate to define the color temperature as that temperature of a black body at which the ratio of the radiant flux at some two arbitrarily chosen wave-lengths, $\lambda_1$ and $\lambda_2$, is the same as that for the radiating substances being investigated. This definition includes the above, but does not demand that the body under investigation should match the black body at all wave-lengths.

Color temperature measurements can be quite accurately made with an ordinary photometer. The emissive powers of the metals that have been studied are larger for the short than for the long wave-lengths; thus the color temperature is higher than the true temperature. Color temperature is to be designated by $T_c(\lambda_1\lambda_2)$. The wave-lengths $\lambda_1$ and $\lambda_2$ are to be given if the temperature refers to two wave-lengths, but when the integral light is used the wave-length may be omitted.

Another "temperature" that has been used in the past but is very little used now is the one that is determined from one of Wien's displacement laws. That is $\lambda_m T = b$. To determine this temperature the wave-length ($\lambda_m$) for maximum emission is determined and the value of $b$, the same as that for a black body, is used and thus a temperature may be computed. No name has been suggested for this temperature.

The fifth temperature that is met with in practice is the one obtained with a total radiation pyrometer from readings of a non-black body. This is called radiation temperature and is to be designated by $T_R$. This temperature is always less than the true temperature.

## II. OPTICAL PYROMETERS

Optical pyrometers, from their construction, are divided into four classes and for convenience of reference are to be classified on this basis.

1. The pyrometer in which the comparison source is a lamp filament located at the focus of the objective lens shall be designated the disappearing filament pyrometer.

2. There does not seem to be any good descriptive name for the pyrometer, of which there are several forms, in which the comparison source is off at a right angle to the direct line of sight and the comparison is made on a photometer cube or some sort of a mirror.

3. The pyrometer, in which the light from the two sources is by some means polarized at right angles to each other and the final balance is made by rotating a nicol in the eyepiece or elsewhere, shall be designated as the polarization pyrometer.

4. All optical pyrometers in which there is no comparison source shall be listed in a separate class and different forms of this class shall be described, if it is found necessary.

The scale for optical pyrometers shall be extended above, or below that of the standard temperature by means of Wien's equation, using for $c_2$ 14350$\mu$ deg. If monochromatic light is used, the wave-length will be obtained from the spectroscope, but if a so-called monochromatic screen is used, the effective wave-length as defined below is to be used.

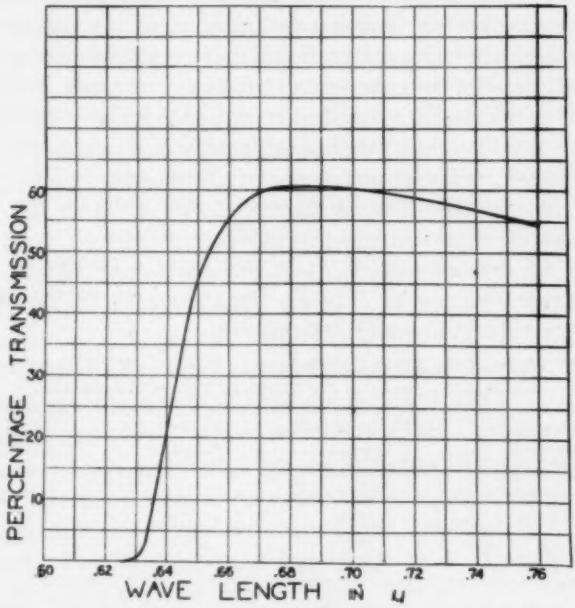



FIG. 1. Spectral transmission of a double thickness (total thickness 10 mm) of Corning high transmission red marked 50% transmission

*Monochromatic Screens.* For the most part it seems best to use as the monochromatic screen in the eyepiece of the pyrometer, a red glass rather than a screen of some other color. It is recommended that for this a red glass be used that has very nearly the same spectral transmission as has two thicknesses (total thick-

ness, 10 mm) of the Corning high transmission red, marked "50% transmission," a spectral transmission curve of which is given in Fig. 1.

If the pyrometer is constructed for temperatures around 1500°K, it is difficult to get enough light for accurate work at very low temperatures. If one of the red glasses is removed from the eyepiece about twice the amount of light will be transmitted, while if both glasses are removed, about fifteen times as much light will reach the eye as with two glasses. It is very convenient to have the eyepiece so constructed that a large sized opening can be used for low temperatures. For extremely low temperatures there will be a very small difference in the readings whether no red glass, one red glass, or two red glasses are used. In Table 1 are given the readings obtained in each case for several temperatures. For the entire range of temperatures given in the Table, the number of red glasses used, i.e., none, one, or two, would make but a small difference in temperature while for the lowest temperature it would make practically no difference.

For the temperature given in the Table a change of .0003 ampere in the current through the pyrometer corresponds to about one degree change in temperature.

TABLE 1. *Current through Pyrometer Filament for Brightness Match for No Red Glass, One Red Glass, and Two Red Glasses for Different Temperatures*

| Temperature degree K | No Monochromatic Screen | One Red Glass | Two Red Glasses |
|---|---|---|---|
| 1404 | .3167 | .3149 | .3148 |
| 1280 | .2786 | .2775 | .2773 |
| 1217 | .2613 | .2604 | .2605 |
| 1141 | .2423 | .2419 | .2422 |

The effective wave-length of the monochromatic screen is defined as the wave-length that is necessary to use to calculate temperatures by means of Wien's equation for brightness measurement with the monochromatic screen in the eye-piece. This is the wave-length for which the same ratio will be obtained using Wien's equation for a certain temperature interval as is obtained by actual measurements from a black body over the same temperature range using the monochromatic screen in question.

*Basis of High Temperatures.* The basis of all high temperature measurements is the work with the gas thermometer[2] that fixed the value of some of the standard melting points. It has often been suggested that the Stefan-Boltzmann law, which states that the total energy radiated by a black body varies as the fourth power of the temperature, be used as the basis of high temperature measurements. This law would give the absolute thermodynamic temperature and would be just as accurate and would depend upon just as sound reasoning as the gas thermometer. In addition, it would be much easier to use the method based upon the fourth power law than the gas thermometer. The container for the gas is the source of difficulty with the gas thermometer, and it seems that there would be very great experimental difficulty with that alone if an attempt were made to extend the gas scale above the present limit reached by Day and Sosman who succeeded in carrying the gas thermometer up to the temperature of melting palladium. The method based on the fourth power could be carried as high as it would be possible to operate a black-body furnace. To be sure, there would be some difficulties with this method, but they do not seem to be insurmountable.

There have been several attempts to compare the scale of temperature as defined by Wien's Equation with that defined by the Stefan-Boltzmann Law. Lummer and Pringshein[3] made a comparison up to 2200°K, but the accuracy was not very high. Gillette[4] has compared the two scales up to 2700°K and states that they are in agreement, but does not state how good the agreement was nor how accurately the two scales could be reproduced. Mendenhall and Forsythe[5] compared the two scales up to 3100°K and obtained the agreement shown in Table 2. These differences are all within the experimental error. The plan is to repeat this comparison under better conditions and to see if the range cannot be extended and also the accuracy increased.

[2] Day & Sosman, Am. Jour. Science, *24*, p. 93, 1910.
[3] Verh. Deut. Phys. Ges. *1*, p. 3, 1903.
[4] Jour. Phys. Chem., *15*, p. 213, 1911.
[5] Phys. Rev., N. S., *4*, p. 62, 1914.

TABLE 2. *Comparison of the Stefan-Boltzmann and Wien Scale*

| Temperature | $T_W - T_{S.B.}$ |
|---|---|
| 2025°K | less than 0.5°C. |
| 2475 | less than +2 |
| 2775 | about +2 |
| 3095 | about +4 |

*Radiation from Non-Black Bodies.* In order to express the relation between the emission of energy from metals and the temperature, a modification of the Wien-Planck equation for a black body is generally used. When the first work was done on the radiation of metals the relation was expressed as

$$E_\lambda = c_1 \lambda^{-a} e^{-\frac{c'_2}{\lambda T}} \tag{1}$$

where an attempt was made to determine $a$ and $c_2$. It was thought for a time that such an equation would represent the facts but later work has shown that $a$ and $c'_2$ are not constant but may be a function of both the temperature and wave-length.

At the present time the modification adopted is

$$E_\lambda = \epsilon_\lambda c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} \tag{2}$$

where the added factor $\epsilon_\lambda$ which is called the spectral emissive power may be a function of both the wave-length and the temperature. According to Kirchhoff's law the emissive power for any wave-length for any body is equal to its absorption factor. If radiation falls upon any body a part is reflected, a part transmitted, and the rest absorbed. The relation between these various quantities is,

$$1 - r_\lambda - t_\lambda = a_\lambda \tag{3}$$

where $t_\lambda$ = transmission, $r_\lambda$ = reflection factor and $a_\lambda$ = the absorption factor. If $\epsilon_\lambda$ is independent of the wave-length the body is said to be gray. As was mentioned above the value of the spectral emissive power for the metals investigated has been found to increase for shorter wave-lengths.

The true temperature and brightness temperature are related as follows:

$$\frac{1}{T} - \frac{1}{S_\lambda} = \frac{\lambda \log \epsilon_\lambda}{C_2 \log e} \tag{4}$$

It has been stated sometimes that for a gray body the brightness temperature is independent of the wave-length. From equation (4) it can be seen that this is not the case, since for a constant brightness temperature $\log e$ must vary inversely as the wave-length. For gray bodies the color temperature is the same as the true temperature.

*Use of an Optical Pyrometer.* It often happens that when it is necessary to measure the temperatures of a hot radiating body with an optical pyrometer the operator does not know just how to proceed. Although optical pyrometers are readily purchasable, if it is necessary to construct one, it will usually be much easier to build one of the disappearing-filament type. All that is required for this is an objective lens about 3 cm in diameter and about 25 cm focal length, a piece of red glass, a long-focus eyepiece, a pyrometer lamp, an ammeter of the right range, a resistance, and a battery.

An ordinary spectacle lens of the right focal length can be used for the objective lens and two such lenses of the proper focal length will do for the eyepiece. For ordinary work it is not necessary to have an achromatic lens since a red glass is used in the eyepiece. The optical parts may all be mounted on some sort of an optical bench or if they are to be used permanently, it is well to have them mounted inside of a tube after the manner of a telescope. The pyrometer lamp is to be so mounted with respect to the objective lens that an image of the source being investigated can be brought to a focus in the plane of the filament and this image and the pyrometer filament both observed through the eyepiece. When using this instrument, the different parts are adjusted so that the images of both the source being investigated and the pyrometer filament are as well defined as possible. The arrangement of the different parts of the disappearing-filament optical pyrometer is shown in Fig. 2. The particular way in which the pyrometer is built depends upon the purpose for which it is intended. If it is desired to measure the temperature of a large source, it will not be necessary to have very large magnification. The distances between the pyrometer lamp and the eyepiece depend upon the focal length of the lens in the eyepiece F,

(in the case illustrated in Fig. 2, a telescope of short focus). For a pyrometer set up to measure the temperature of small filaments the dimensions given below Fig. 2 have been found satisfactory. The main requirement is for the image of the source in the plane of the pyrometer filament to be much larger than the pyrometer filament, since it has been found by experiment that accurate matches cannot be made unless the background image is somewhat larger than the pyrometer filament. Due to diffraction, there will be dark or bright streaks along the edge of the pyrometer filament, and these streaks will be observed if an eyepiece

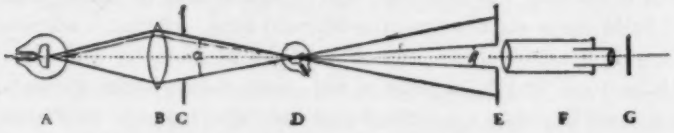



FIG. 2. Diagram showing arrangement of apparatus. A, background; B, objective lens; C, entrance cone diaphragm; D, pyrometer filament; E, eyepiece diaphragm; F, eyepiece; G, monochromatic filter. (AB=25 cm; BD=128 cm; DE=185 cm; diameter of aperture at E=9 mm)

with sufficient resolving power is used. If they can be seen, they interfere with the accuracy of the setting. In order to avoid this difficulty, reduce the resolving power of the eyepiece, either by using an eyepiece in the telescope with a smaller resolving power or using a cap with a small hole over the objective lens of the eyepiece telescope.

With an optical pyrometer the intensity of the light per unit area, that is, the brightness, for a particular wave-length interval, is compared with the standard. To calibrate the pyrometer to measure temperatures, it is necessary to find a relation between this brightness and the temperature. Such a relation is given by Wien's equation which shows the relation between the energy for a particular wave-length interval, the wave-length, and the temperature. Thus, if the wave-length is known for which the energy, $E_\lambda$, is compared, the temperature can be calculated. $E_\lambda$ is compared with that of the standard source either by a spectrometer, which determines the wave-length, or by the use of a particular monochromatic screen in the eyepiece. In the former case, if proper precautions are taken to have slit-widths of the

proper size, the wave-length can be definitely determined. When a so-called monochromatic screen (usually a red glass) is used, its effective wave-length must be known. This effective wave-length defined above is slightly different for different temperature intervals, and is the wave-length that must be used for the different temperature intervals so that calculations may be made.

*Calibration of Optical Pyrometers.* Despite all that has been written about the calibration of an optical pyrometer, a question is very often asked concerning the starting point for the calibration of the pyrometer. To make a completely independent calibration of any pyrometer would be a very great undertaking. Very fortunately it is not necessary or desirable that each one should make an absolutely independent calibration. If one had to calibrate an ordinary mercury thermometer at about 100°C, it would be calibrated by immersing the thermometer into steam. If a higher point was wanted some material with a melting or boiling point in the proper region might be used, and the calibration made at the melting or boiling point as the case might be. The temperature of boiling water is defined as 100°C but the temperature of the other points in this region are used with just as little question and with just about as much confidence as this one. To probably a lesser extent the same condition exists at high temperatures. The temperature of either melting gold or palladium is very constant and is well known. It only remains to bring the standard radiator, that is the black body, to either one of these definite temperatures and from this to calibrate the pyrometer. In this case, the pyrometer filament is matched in brightness with the image of the black body across which it is seen through the eyepiece of the pyrometer. How this is done, together with a description of the black body, is given in the next few paragraphs.

*Black Body.* The Kirchhoff ideal black body consists of a cavity having uniformly heated walls with an opening into the cavity, through which the radiation is studied. It has been shown that if this opening is very small with respect to the size of the walls of the cavity, the error that will be introduced by this opening will be negligible. Many different attempts have been made to realize

in a practical way the black body. The first attempts were made by surrouding variously shaped cavities by molten materials. This method enables a cavity to be heated quite satisfactorily. A long tube uniformly heated gives very good black-body conditions. By inserting a few diaphragms in this uniformly heated tube with a small opening at the end, it was found to give quite satisfactory black-body conditions. Such tubes can be made from some suitable refractory materials wound with either a wire or ribbon resistor. If the black body thus wound is intended for use at the temperature of the gold point (1336°K) a resistor made of nichrome wire or ribbon is found to work quite well. However, for operating in the open air for any length of time at temperatures much higher than this the only satisfactory material from which the resistor can be made is platinum. For this work, in general, platinum ribbon about 2 cm wide and about 0.01 mm thick is used. A description of a black body that has been found quite satisfactory has been given in the Astrophysical Journal, Vol. 51, p. 247, 1920.

When a furnace is used as a standard of radiation for calibration of an optical pyrometer, it is held at either the temperature of melting gold or melting palladium. In order to be sure that the furnace is at the temperature desired, a sample of the metal is melted inside the furnace. To do this the sample of gold or palladium is mounted between platinum wires supported by two refractory tubes and the circuit completed through a battery and bell or other indicating instrument. It is necessary to mount the specimen in such a manner that the central part of the sample cannot touch either of these two tubes, as it has sometimes been found that if a single tube is used the melted metal will touch and cling to the end of the tube so that the electric circuit is not broken at the instant of melting and thus a high value is obtained. With the central part of the sample free, it is assumed that the circuit is broken at the instant of melting. When a melt is obtained, the furnace is held at this temperature, using as an indicator the emf of the thermocouple, and then a number of readings made with the optical pyrometer sighted upon the black body. Another melt is then obtained and the process repeated. This

procedure is necessary because it has been found that the emf of a platinum platinum-rhodium thermocouple, unless the most exceptional precaution to eliminate impurities in its manufacture and to prevent contamination during use, have been taken, changes constantly when used at the high temperatures.

The standard radiator at the standard temperature gives one point on the calibration. To obtain another point,[6] take readings of the current through the pyrometer filament for an apparent brightness match with a rotating sector, or absorbing glass of known transmission, between the pyrometer lamp and the standard black body. This will give a measure, in terms of the pyrometer current, of a brightness that is some known fraction of that of the standard radiator at the standard temperature. If monochromatic radiation is used, it is easy to calculate the temperature $T_2$ of the black body corresponding to this current through the pyrometer filament, that is to this measured brightness, from $T_1$ the standard temperature by the following formula derived from Wien's equation:

$$\frac{1}{T_1} - \frac{1}{T_2} = \frac{\lambda \log R}{c_2 \log e} \tag{5}$$

where $R$ is the transmission of the sector, and $\lambda$ the wave-length used. If the measurements are made with a red glass in the eyepiece, the temperature that would correspond to this fraction of the brightness of the standard black body can be calculated just as before excepting that in this case the effective wave-length of the red glass for the temperature interval is to be used.

*Extending Temperature Scale.* If a rotating sector or an absorbing glass of known transmission is used between the source being investigated and a calibrated pyrometer lamp, temperatures higher than that of the standard black body can be measured. In this case, the temperature is to be calculated from the temperature corresponding to the current through the pyrometer filament and the transmission of the sector or absorbing glass used by means of equation (3) above, excepting in this for $R$, use the reciprocal of the transmission of the sector or glass, and then $T_2$

[6] Mendenhall, Phys. Rev., *33*, p. 74, 1911.

will come out greater than $T_1$. A very convenient method is to work out such extrapolated temperatures for the various sectors and absorbing glasses that are to be used, and plot the extrapolated temperatures against the temperatures as determined from the pyrometer reading. Such curves can then be used with any pyrometer using the same red glass, providing the same sectors or absorbing glasses are used.

Wien's equation is generally used to calculate temperatures from brightness measurements. This equation does not fit the experimental evidence excepting for small values of the product $\lambda T$. Table 3 has been prepared to show just what errors in both the true and color temperature are obtained in the use of this equation rather than Planck's more exact form. The results for true temperature were obtained by calculating the higher temperature from 1828°K and wave-length of $\lambda = 0.665\mu$.

For color temperature the values given were obtained by calculating the color temperature from the ratio of the red ($\lambda = .665\mu$) to the blue ($\lambda = .467\mu$).

TABLE 3. *Corrections to be added to Temperatures obtained from Wien's Equation to Reduce them to what would be obtained from Planck's Equation*

| Temperature Degree K | Corrections to | |
|---|---|---|
| | True Temperature | Color Temperature |
| 2000 | −>0.01 | >.01 |
| 2600 | − 0.05 | .1 |
| 3000 | − 0.3 | .7 |
| 3600 | − 1.5 | 3.2 |
| 5000 | − 15.6 | 30.7 |

From Table 3 it can be seen that Wien's equation is quite satisfactory for temperatures up to about 3600°K, since the error obtained by its use is less than the experimental error. It is to be noted that the error obtained in color temperature is about twice the magnitude of the error in the brightness temperature and in the opposite direction.

It has been found in photometry that two adjacent surfaces can be brought very accurately to the condition of the same brightness by eye observations if there is no visible line separating

them. If the disappearing-filament pyrometer is properly lined up and the proper magnification and resolving power is used, there will be no visible line separating the pyrometer filament from the image of the background, thus the conditions for brightness matches are good. If properly adjusted the polarization type of pyrometer has a very good field since the line separating the two parts disappears entirely. Other types of pyrometers can be so constructed that the line of separation between the two parts is quite satisfactory.

## DEPENDENCE OF THE TEMPERATURE ON VARIOUS CONDITIONS

(a) *The Constant* $c_2$. As was noted in the 1920 report, the value of $c_2$ that then seemed to fit all the experimental data is $14350\mu$ degrees. Later data indicate that the value of $c_2$ should be reduced somewhat. Coblentz gives as the best value $14\,320\mu$ deg. In calculating temperature from brightness comparison the results obtained depend upon the value of $c_2$ used. It is often necessary for purpose of comparison to change the values of a particular temperature to that which would have been obtained if a different value of $c_2$ had been used. Temperatures are in general calculated from a standard temperature using Wien's equation. From this equation the following relation is obtained between two temperatures, $T'_2$ and $T_2$ obtained by use of two different values for $c_2$, from an initial temperature $T_1$, and from a particular ratio of brightness and wave-length $\lambda$:

$$\frac{1}{T_2}-\frac{1}{T_1}=\frac{c'_2}{c_2}\left(\frac{1}{T'_2}-\frac{1}{T_1}\right) \tag{6}$$

Using this equation with $T_1=1336°K$ (gold point) the corrections were calculated for various temperatures and various values of $c_2$. These corrections have been plotted as shown in Fig. 3. From these curves a temperature corresponding to a particular $c_2$ can be corrected to any desired value of $c_2$. As an illustration of the use of this set of curves, a correction will be obtained. Suppose that a temperature of 2500°K is found for $c_2=14\,350$ and the correction is desired for $c_2=14200$. From the curve marked 14 200, the correction for 2500°K is seen to be $+25°$. Thus the temperature should be 2525 for $c_2=14200$.

(b) *Brightness.* A very great advantage of the optical pyrometer in measuring temperature is due to the relation between a change in the brightness and the corresponding change in temperature.

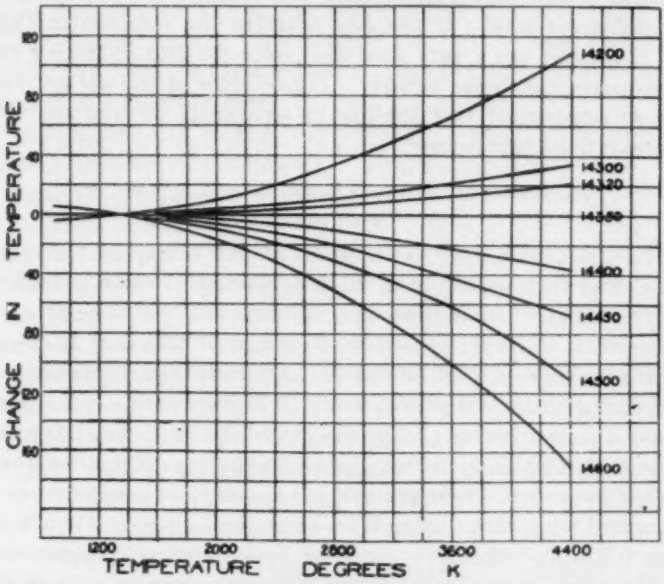



FIG. 3. Curves showing differences obtained in calculating temperature from $T_1$=1336°K when different values of $c_1$ are used

At 1000°K for red radiation, ($\lambda = 0.665\mu$) the brightness varies about 22 times as fast as the temperature. At 2000°K for red radiation the ratio is about 11. From Wien's equation the following is the relation between the change in energy and the change in temperature:

$$\frac{dE_\lambda}{E_\lambda} = \frac{dT}{T}\frac{c_1}{\lambda T} \tag{7}$$

The relation between a change in temperature and the corresponding change in brightness for the red ($\lambda = 0.665\mu$), green ($\lambda = 0.57\mu$) and blue ($\lambda = 0.475\mu$) has been computed for temperature ranges, and the results platted as shown in Fig. 4. From these curves the ratio of the change in brightness to the change in temperature for any of the three wave-lengths and any tempera-

ture can be found. For example at a temperature of 2500° for $\lambda = .57$, the brightness changes about ten times as fast as the temperature.

When measuring the temperature of a source with an optical pyrometer, there are some precautions that should be taken. Very fortunately the maker of the pyrometer has been able to take care of some of these in the construction of the instrument.

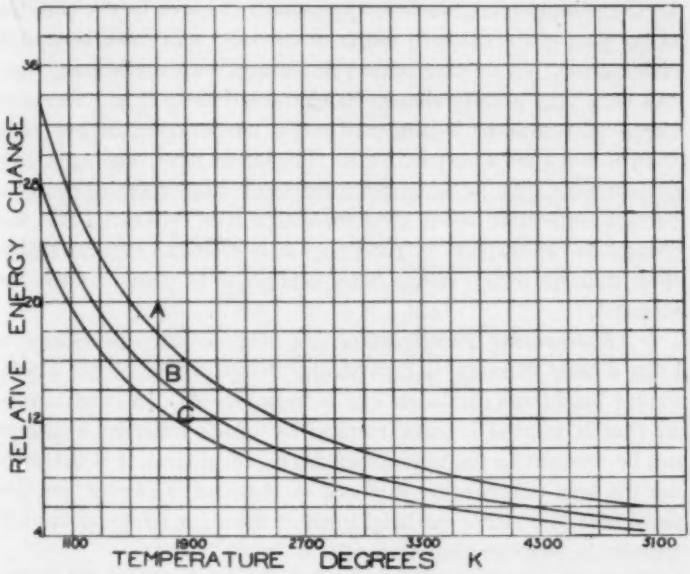



FIG. 4. Curves showing the relative change in energy for percent change in temperature curve A, for Blue ( $\lambda$=0.475 ); curve B for Green ($\lambda$=0.57$\mu$); curve C for Red ( $\lambda$=0.665 )

(c) *Clean Optical Parts.* The first thing to keep in mind is that it is a calibrated instrument and must be handled as such. All the optical parts of the instrument should be kept clean. The actual error due to dirt on the objective lens is not very great. A lens has to be very dirty to cause an error of 10° at 1500°K. However, errors much larger than this are possible if the lenses are allowed to become too dirty. An error of 15° at 1500°K would be caused by allowing the lens to become so covered with dirt that its transmission would be reduced to 85% of its value when clean. If the lenses and pyrometer lamp bulb are allowed to become

dirty, it interferes with the clearness of the images and thus lessens the accuracy of the setting. They should be kept clean but in cleaning them care is required or scratches will be produced which will again interfere with the definition.

(d) *Comparison Source.* Very great care must be taken not to change the comparison source in any way. In some pyrometers even the position of the comparison lamp must not be changed. In the disappearing-filament type small changes in the position of the part of the filament under observation will cause no appreciable error. In no case must the filament be over-heated, since this may very greatly change its characteristics. The pyrometer lamp, of tungsten, if not used at a temperature higher than 1828°K will have a very long life. The lamps have been aged for a considerable time at a temperature much higher than this, so a current somewhat above that for which it is calibrated will not change its calibration. A good plan is to allow no current higher than that for which it has been calibrated to pass through the filament.

(e) *Reproducing Temperatures.* It is quite often necessary to bring a lamp filament to a particular temperature so that a particular brightness condition can be reproduced. In many cases, all that is wanted is some method by which different filaments can be brought to the same brightness. In general, it is better to use the true temperature if it can be obtained; however, for this particular case either the brightness or the color temperature will answer the purpose quite well.

The brightness temperature has been defined above. The color temperature is defined as the temperature of a black body at which it has in general the same integral color as the source being investigated. These color matches can, generally, be made very easily and very accurately with an ordinary contrast Lummer-Brodhun photometer. To do this the method of substitution is used, the lamp being studied first put in place and set at the desired voltage. The resistance that controls the current through the comparison lamp is then changed by small amounts, the observer at the same time keeping an intensity match by moving either the photometer or one of the lamps. By thus changing the comparison lamp the two lamps can be brought very accurately to the same color. The unknown lamp is then removed, and the

standard put in its place, and now the comparison kept at the voltage previously determined, and the voltage of the standard changed until there is again a color match. It is to be noted that there is to be an intensity match at the same time that a color match is obtained, since it is impossible to set for color match unless there is at the same time a very good intensity match.

Such color matches can be made for the various filament lamps and the black body and most of the flame standards of light. Such color matches are impossible between any of the above standards and the Welsbach gas mantle or any source that does not have a continuous distribution of energy through the visible spectrum.

TABLE 4. *Relation between Lumens per Watt and Color Temperature for a Tungsten Lamp*

| Lumens per Watt | Color Temperature |
|---|---|
| 0.5 | 1644 |
| 1.0 | 1777 |
| 1.5 | 1866 |
| 2.0 | 1939 |
| 2.5 | 1998 |
| 3.0 | 2050 |
| 3.5 | 2096 |
| 4.0 | 2138 |
| 4.5 | 2175 |
| 5.0 | 2208 |
| 5.5 | 2241 |
| 6.0 | 2269 |
| 6.5 | 2299 |
| 7.0 | 2327 |
| 7.5 | 2354 |
| 8.0 | 2380 |
| 8.5 | 2406 |
| 9.0 | 2431 |

To obtain the color temperature, it is necessary to have a standard of color temperature,[7] that is, a lamp that has been compared in color with the black body at various temperatures. The relation between color temperature and lumens per watt has been very carefully worked out for a wide range of temperatures. It has been found that this relation holds very well for different lamps of about the same type. In Table 4 are given these relations for tungsten for a range of efficiencies from 0.5 lumens per watt to 9.0 lumens per watt.

[7] Hyde, Cady, and Forsythe, Phys. Rev. N. S., *10*, p. 395, 1917.

These values are for the ordinary type of vacuum lamps. Thus they would not hold if applied to a gas-filled lamp. If a standard of color temperature is not available, quite accurate results can be obtained by measuring the lumens per watt and applying the above relations.

## III. RECENT LITERATURE ON PYROMETRY

J. A. Anderson[8] at the Mt. Wilson Observatory reached a very high temperature by electrically exploding very small wires. He exploded these wires by discharging through them a condenser of large capacity. According to his estimate the brightness reached corresponded to a temperature of about 20,000°K.

In September 1919, there was held in Chicago, in connection with the American Institute of Mining and Metallurgical Engineers, a Symposium on Pyrometry. The papers and discussions given at this Symposium have been published in book form. In this book there are to be found about sixty papers that cover almost the entire field of pyrometry. While there is some overlapping in the ground covered, this is not of such a nature as to decrease the value of the book.

Priest,[9] describes an interesting instrument called the leucoscope. With this instrument, he is able to construct an apparently quite accurate pyrometer without the use of any comparison source. From the description of the measurements made by Mr. Priest, it seems that he has uncovered quite a valuable instrument for various investigations in the study of color.

Technologic Paper (No. 170) of the Bureau of Standards by Foote, Fairchild, and Harrison, entitled "Pyrometric Practice," a book of 326 pages and 187 figures, gives quite a complete survey of the field. In this are found descriptions of methods of calibrating the various forms of pyrometers, together with the methods for their use.

LABORATORY OF PURE SCIENCE,
NELA RESEARCH LABORATORIES, CLEVELAND, OHIO,
October 1, 1921.

[8] Astrophysical Journal, *51*, page 37, 1920.
[9] JOURNAL OF OPTICAL SOCIETY, 4, p. 448, 1920.

# SUPPLEMENTARY NOTE ON THE FREQUENCIES OF COMPLEMENTARY HUES

BY

IRWIN G. PRIEST

Referring to the "Note on the Relation Between the Frequencies of Complementary Hues,"[1] Dr. Selig Hecht has kindly pointed out in a letter to the author, that, if the relation between frequencies is hyperbolic, it follows of mathematical necessity that the relation between wave-lengths is also hyperbolic. This is true. It might therefore be thought a matter of indifference as to whether frequency or wave-length is considered in presenting such data. There is, however, another aspect of the question: *If frequency scales be used, the experimental data fall symmetrically about the vertex of the hyperbola.* (J. Op. Soc. Am., *5*, p. 404, Fig. 2). *If wave-length scales be used, two-thirds of the same data fall on one side of the vertex.* (J. Op. Soc. Am., *5*, p. 404, Fig. 1.)

It appears therefore that, as regards the treatment of experimental data, the relation can be more conveniently and accurately demonstrated on frequency scales. These are the points worthy of notice. The author regrets that in emphasizing them he was led to imply the contradiction of a mathematical theorem.

It may be noted, that the asymptotes of these hyperbolic equations correspond roughly to the "neutral points" in the spectra of the "red-green blind" and the "yellow-blue blind" respectively,[2] thus:

In vibrations per trillionth of one second,

| Asymptotes of Hyperbola | Neutral Points |
|---|---|
| 530 | 522 yellow-blue blind (not well known) |
| 608 | 612–601 red-green blind. |

[1] J. OPTICAL SOC. AM., *5*, p. 403; Sept. 1920.

[2] Koellner: Stoerungen des Farbensinnes, pp. 42 and 49.

In words, the spectral region lying between the "neutral points" for these two general classes of "color blinds" is approximately coextensive with the region for which the normal eye finds no spectral complementary.

BUREAU OF STANDARDS

Nov. 5, 1921.

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